

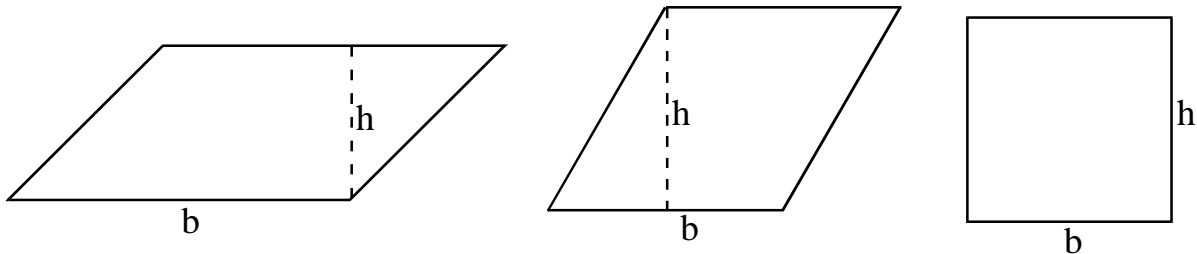
Area

Geometry 8.1

Rectangles, Parallelograms, Triangles, Trapezoids, Kites:

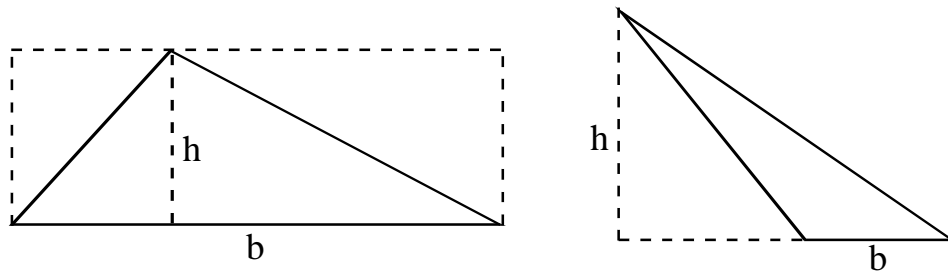
1. What formula can you use for the area of any parallelogram (including rectangles, squares, and rhombuses)?

Diagram why this formula will work even for the 'tilted' shapes.



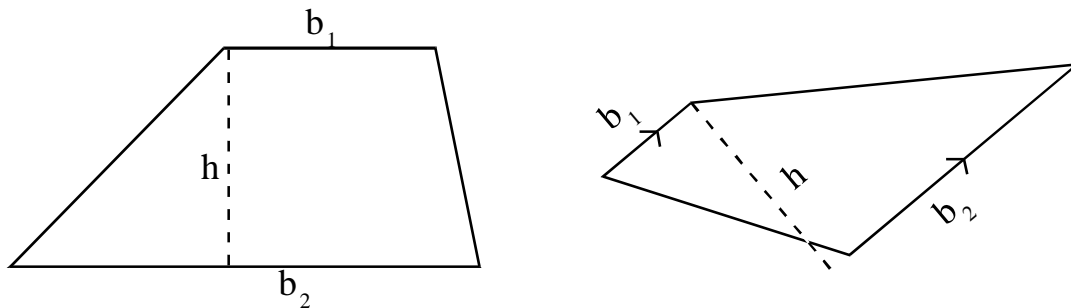
2. What formula is used to determine the area of a triangle?

Explain why it always works.



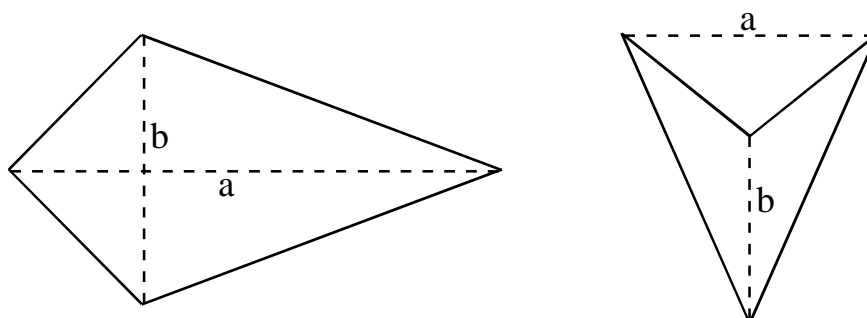
3. What is the formula for the area of a trapezoid?

Can you create a diagram which demonstrates why this formula works?



4. What is the formula for the area of a kite?

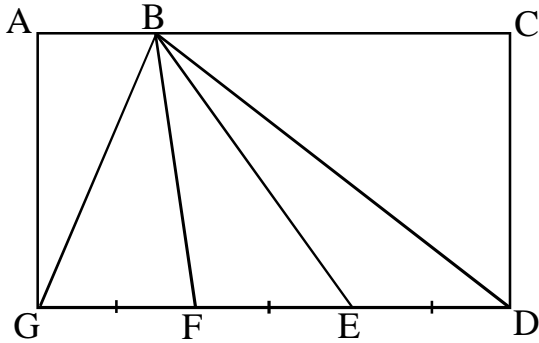
Can you explain why this formula works?



Area

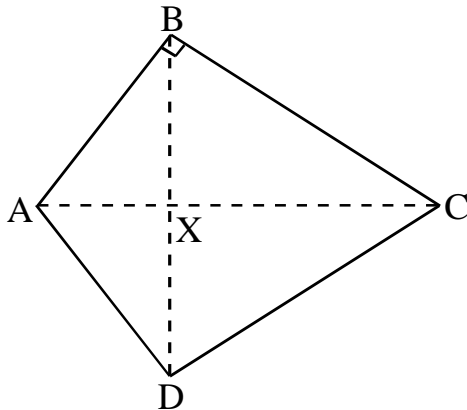
Find the area of each:

Rectangle ACDG has an area of 90cm^2 .



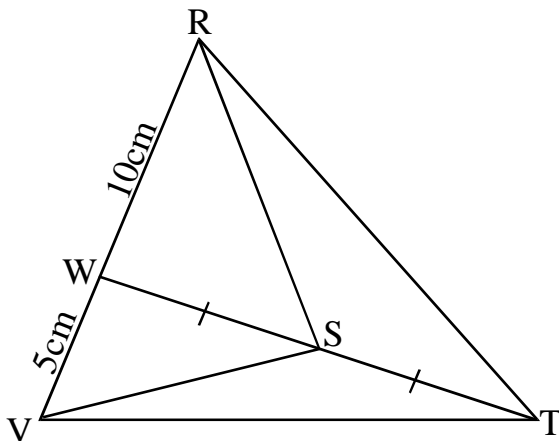
1. What is the area of triangle BGD?
2. What is the area of triangle BFD?
3. $AB=4$, $AG=6$. Find the area of trapezoid BCDF.

ABCD is a kite (not drawn to scale). $AC=50$, $BD=48$, $AB=30$.



1. What is the area of kite ABCD?
2. What is the length of BC?
3. Find the area of triangle ABX.

Triangle VRT has an area of 180cm^2 (not to scale).

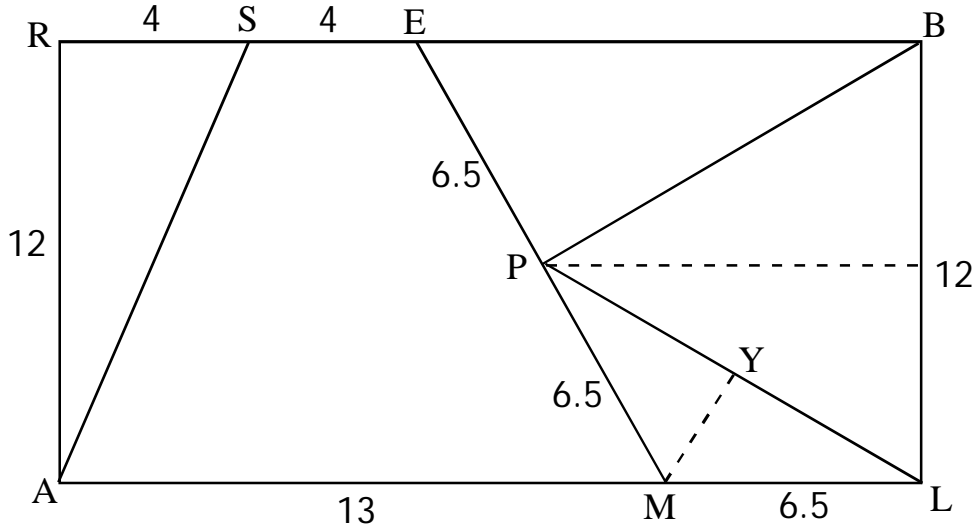


1. What is the area of triangle RWT?
2. What is the area of triangle RST?
3. What is the area of triangle VTS?

Area Practice

Geometry 8.2

Find the area of each figure listed:

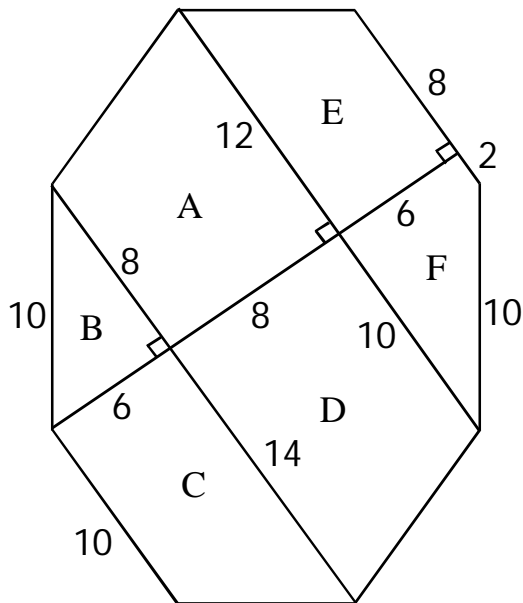


1. Triangle SAR: _____
2. Trapez. MASE: _____
3. Triangle PEB: _____
4. Triangle BPL: _____
5. Triangle PML: _____
6. Pent. BRALP: _____
7. Trap. BEML: _____

Answer: Explain each on the back of this sheet.

8. Is the area of all the interior figures equal to the area of rectangle RALB?
9. Is BLMP a kite?
10. Determine the length of segment PL.
11. Can you determine the length of segment MY?

Find the area of each lettered piece:



12. What is the area of the octagon?

13. Label the length of the missing sides of the octagon.

Mass Points

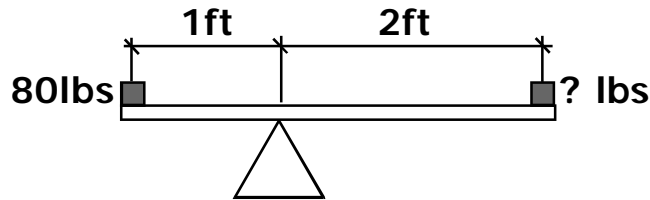
Geometry

Mass Points Geometry

I have been looking for a good place to include this lesson and this seems like as good a place as any ... it was developed by HIGH SCHOOL STUDENTS in the 60's for use in math competitions as a 'trick' to simplify complex problems.

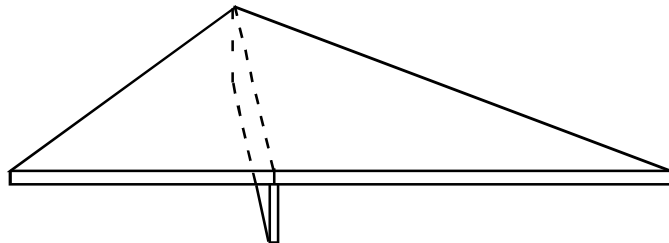
Consider the following see-saw problem:

What mass of the box on the right is required to balance the box on the left?

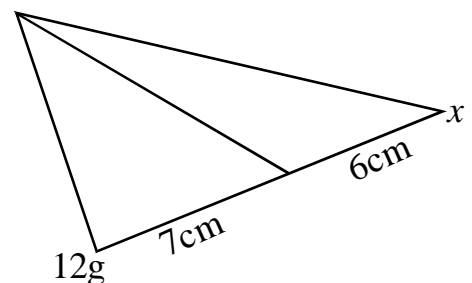
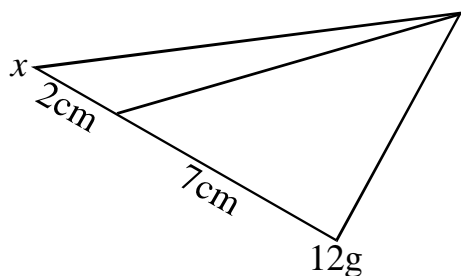
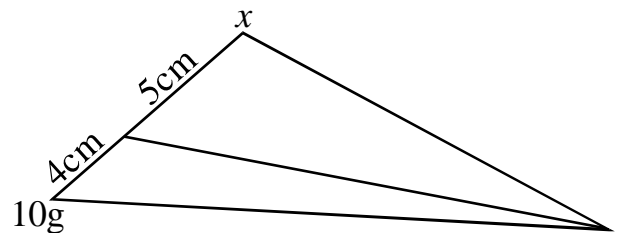
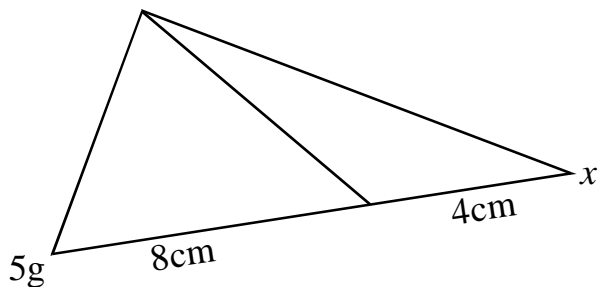


Consider a similar problem:

Think of the see-saw as a big metal triangle of uniform thickness. The same principal holds true. If we place a weight on the left and a weight on the right, mass times distance must be equal on both sides. Try a few examples.



Practice: Find the mass in grams x at each vertex.

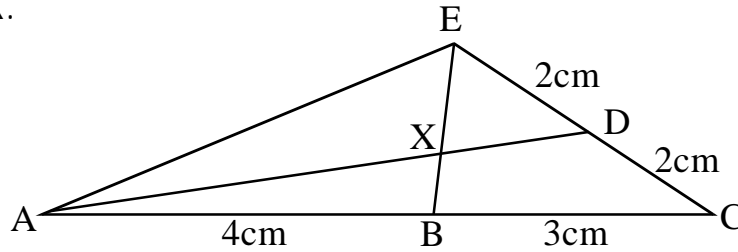


Mass Points

Now for some complexity ... but we will stick to the simplest cases where mass point geometry is useful.

Example. Try the following:

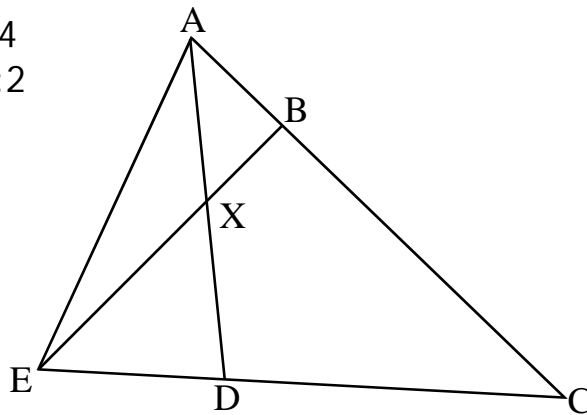
Find $AX:DX$ and $EX:BX$.



Practice:

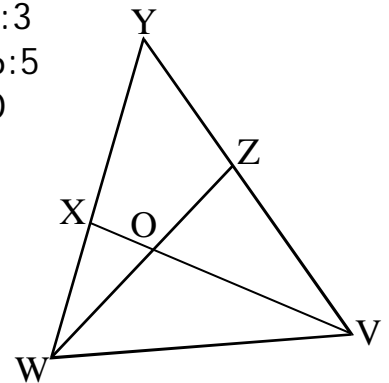
1.

$AB:BC = 1:4$
 $ED:DC = 1:2$
 Find $EX:BX$
 and $AX:DX$



2. (harder)

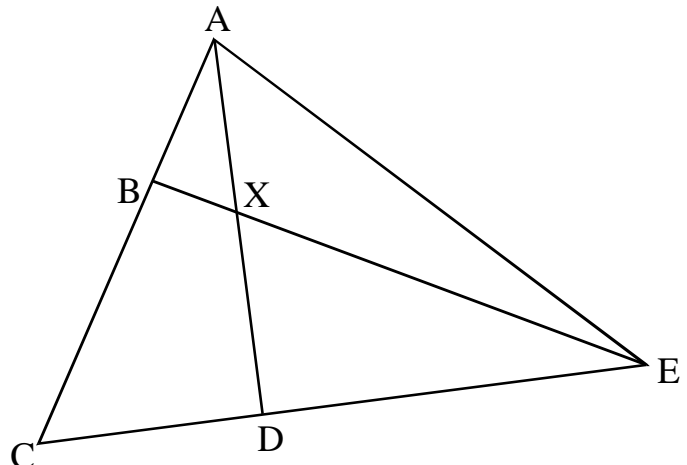
$WX:YX = 2:3$
 $WO:ZO = 6:5$
 Find $XO:VO$
 and $YZ:VZ$



So, what does this have to do with area?

The area of triangle ABC is 36cm^2 .
 If $BC=2AB$ and $CD:DE = 3:5$, find
 the area of each polygon listed:

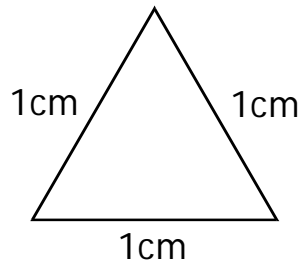
1. Triangle ABE: _____
2. Triangle ADE: _____
3. Triangle AXE: _____
4. Triangle DXE: _____
5. Triangle ABX: _____
6. Quadrilateral BCDX: _____



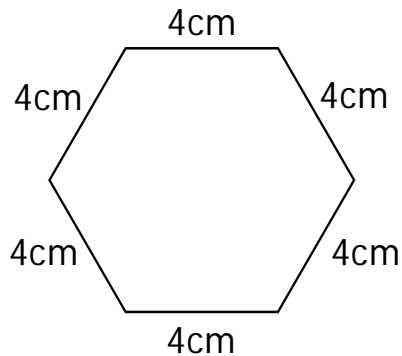
Area of Regular Polygons

Geometry 8.4

How could you determine the area of the triangle below?



Use similar strategy to determine the area of the hexagon below:



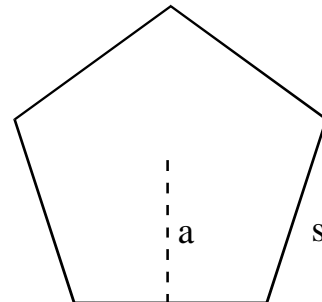
The two special cases above are easier because the height of the triangle can be determined using the Pythagorean Theorem. For most polygons, this is not possible without using trigonometry.

What formula could be used to determine the area of a regular polygon given the:

Number of sides: n

Side length: s

Apothem (inradius): a



How can this formula be simplified given the perimeter P of the polygon?

Find the area of each regular polygon below:

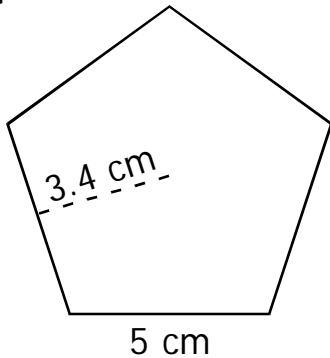
1. A nonagon whose side length is 12cm and whose apothem is 16.5cm?
2. A polygon whose perimeter is 60 inches and whose apothem is 8.5 in?

Area of Regular Polygons

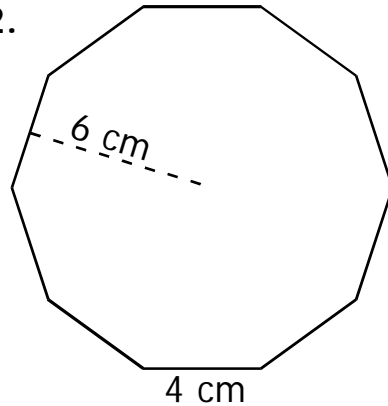
Geometry 8.4

Determine the area of each figure below:

1.



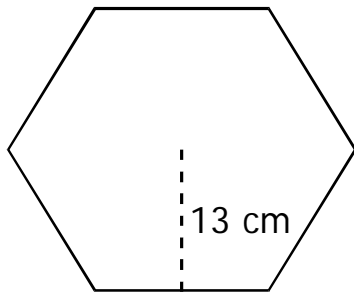
2.



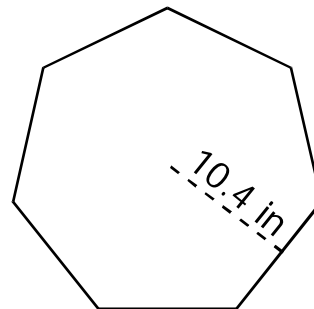
What is the perimeter of each figure below?

(round to the tenth)

1. Area = 585m^2



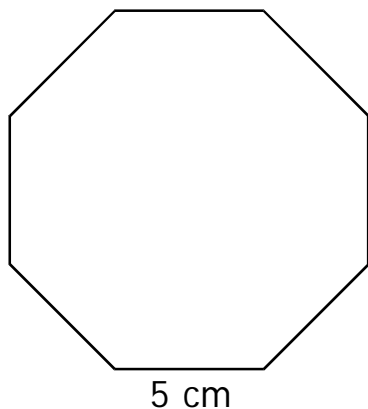
2. Area = 364in^2



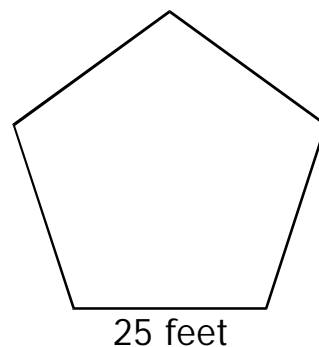
What is the apothem of each regular polygon below?

(round to the tenth)

1. Area = 121cm^2



2. Area = 1075ft^2

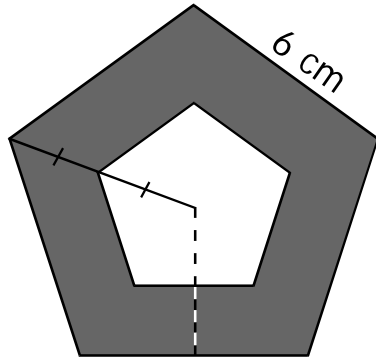


Area of Regular Polygons

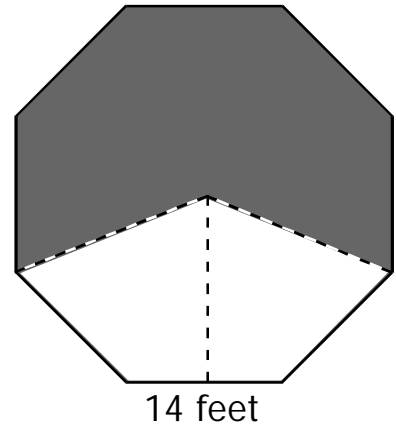
Geometry 8.4

Determine the area of each shaded figure below:

1. $a = 4.4$ cm
 $s = 6$ cm

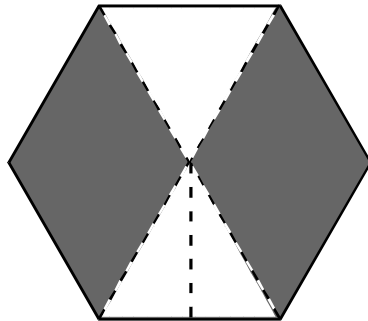


2. Round to the tenth.

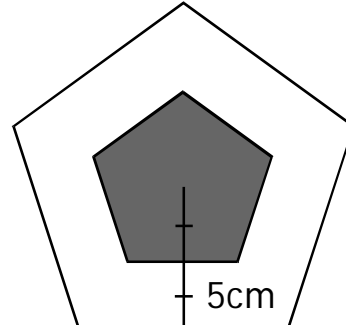


What is the outside perimeter of each figure below?
(round to the tenth)

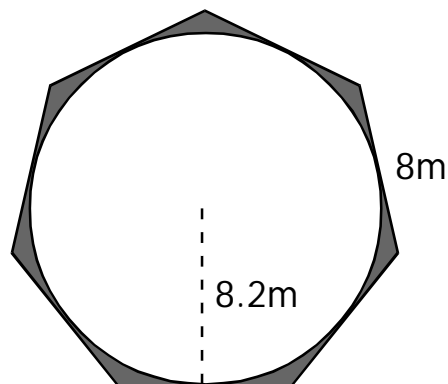
1. Shaded Area = 170m^2
 $a = 8.5$



2. Shaded Area = 91 in^2



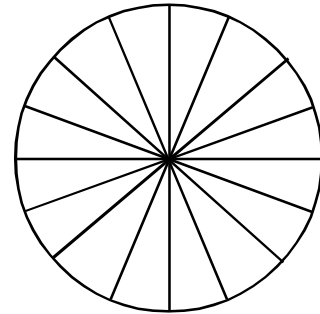
Find the area of the shaded region below:
Round to the tenth.



Area of Circles

Complete the activity below to explain why the area of a circle is equal to pi times the radius squared:

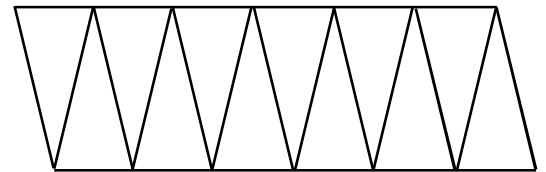
Sketch a circle sliced into many pieces.



A circle is essentially a polygon with infinite sides of infinitesimal length.

We can therefore imagine each slice as a triangle.

Those triangles can be rearranged into a parallelogram.

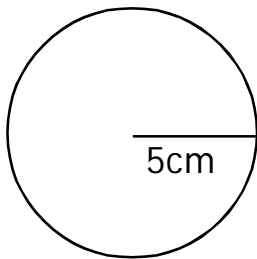


Use the dimensions of the parallelogram to 'discover' the formula for circle area.

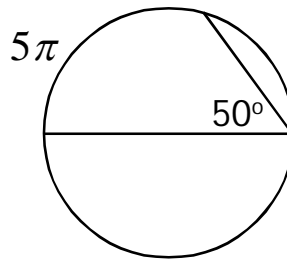
Solve

Find the area of each circle in terms of pi:

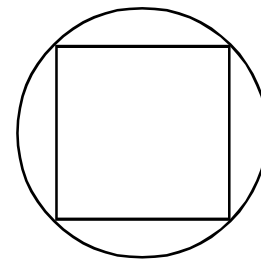
1.



2.



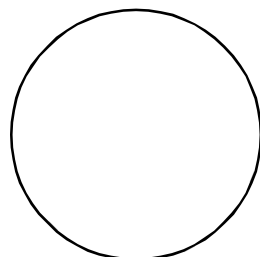
3. Square = 1cm²



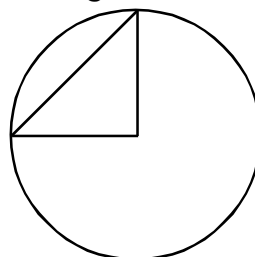
Solve

Find the circumference of each circle:

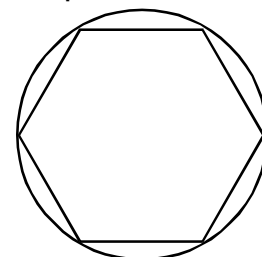
1. Area = 4π



2. Triangle = 8cm²



3. Hex perim. = 24cm

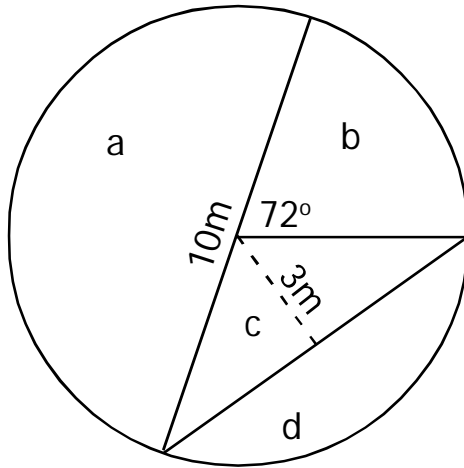


Area of Circle Parts

Determine the area of each labeled region below:

Express in terms of pi and/or rounded to the tenth.

1.



half-circle a = _____m²

sector b = _____m²

triangle c = _____m²

circle segment d = _____m²

Vocabulary:

A segment is the region between a chord and the included arc.

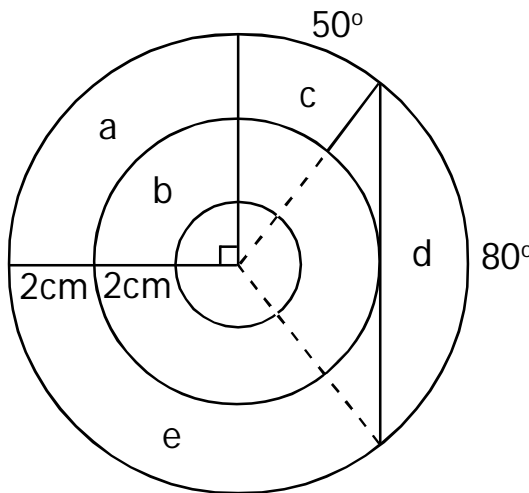
A sector is a 'slice' between two radii.

An **annulus** is the region between concentric circles (see below).

Determine the area of each labeled region below:

Express in terms of pi and/or rounded to the tenth.

The diameter of the small inner circle is 4cm.



piece a = _____cm²

piece b = _____cm²

piece c = _____cm²

piece d = _____cm²

challenge piece e = _____cm²

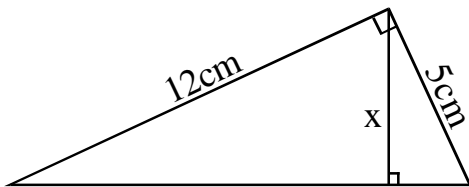
Other Uses for Area:

Geometry

Finding an altitude:

If you know the area of a figure, you can often find the length of an altitude.

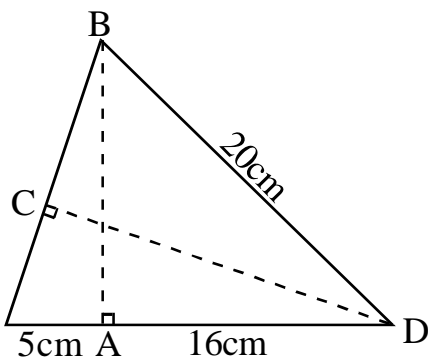
Example: Find the length of x in the right triangle below. Express your answer as a common fraction.



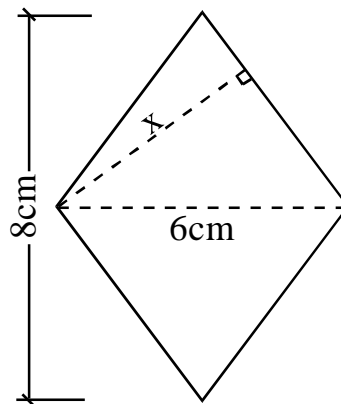
Hard Practice:

Find each missing length.

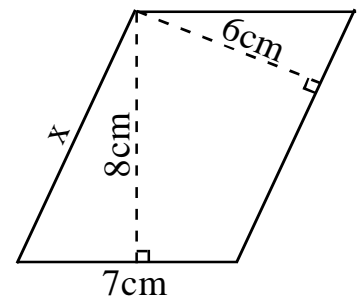
1. Find Altitude CD .



2. Rhombus



3. Parallelogram



Word Problems:

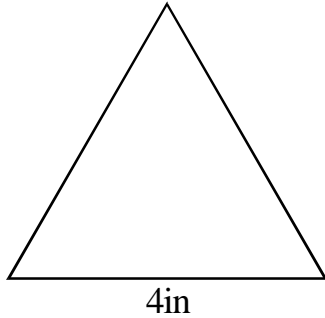
1. The area of a parallelogram is 400cm^2 . If the sides of the parallelogram measure 16 cm and 40 cm, what is the length of the long altitude?
2. What is the altitude to the hypotenuse of a right triangle whose legs measure 20 and 21 cm?
3. The short diagonal of a rhombus of side length 25 cm is 14 cm. Find the diameter of the inscribed circle in the rhombus. Express your answer as a mixed number in simplest form.

Using Pi and Radical Notation

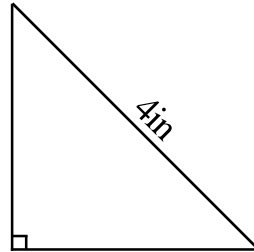
Geometry 8.7

Practice: Find the area of the following triangles.
Leave answer in simplest radical form:

1. (equilateral)

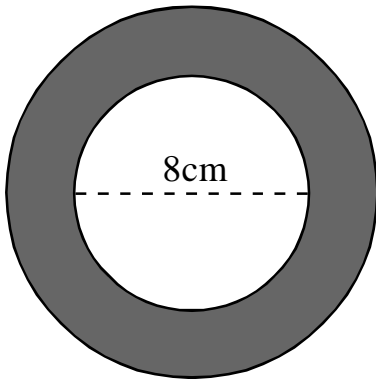


2. (isosceles right)

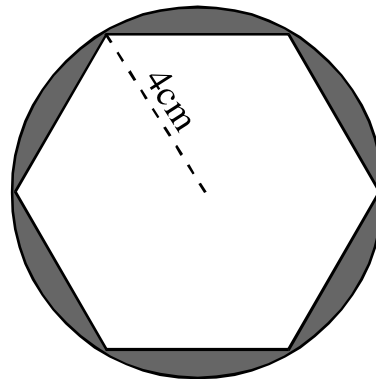


Practice: Find each shaded area. Leave answers in terms of Pi or in radical form.

1. (large radius = 7cm)



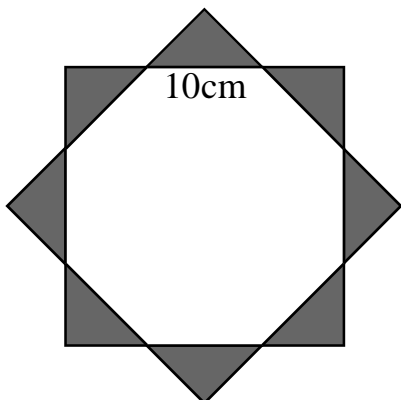
2. (hexagon is regular)



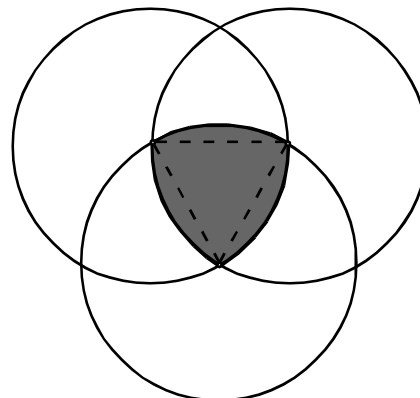
Find each shaded area.

Leave answers in simplest radical form and/or in terms of Pi.

1. (octagon is regular)



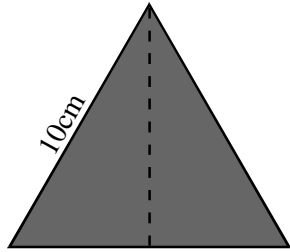
2. (congruent circles have 6in radii)



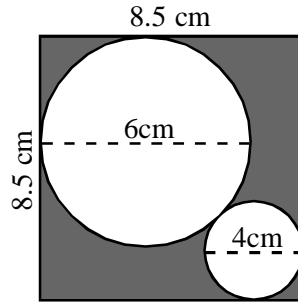
Using Pi and Radical Notation

Geometry 8.7

Practice: Find each shaded area. Answers should be in simplest radical form and/or in terms of Pi where applicable.

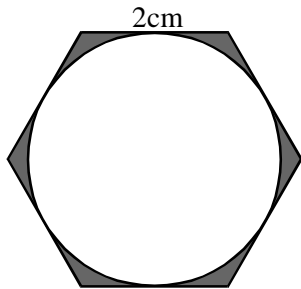


1. _____

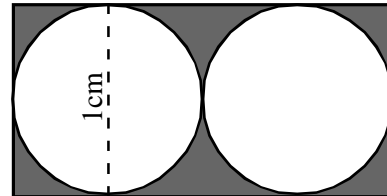


note:
8.5 is a close approximation.
Can you find the real side length of the square in this diagram?

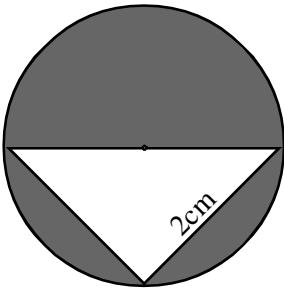
2. _____



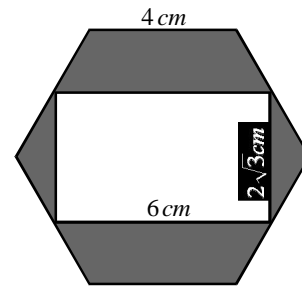
3. _____



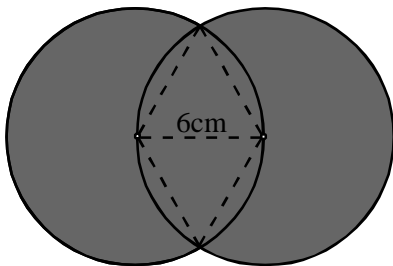
4. _____



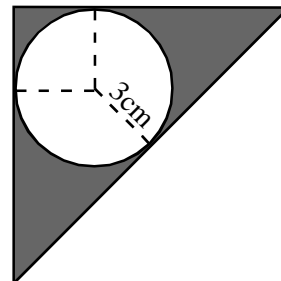
5. _____



6. _____



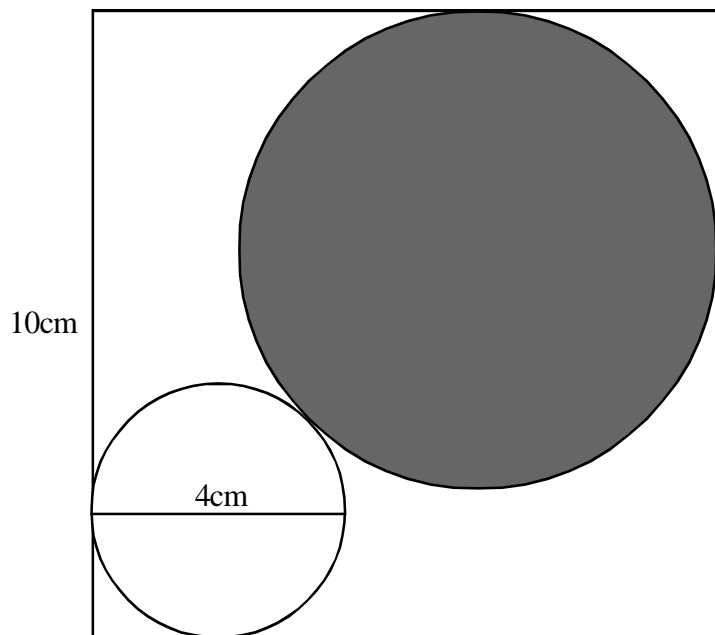
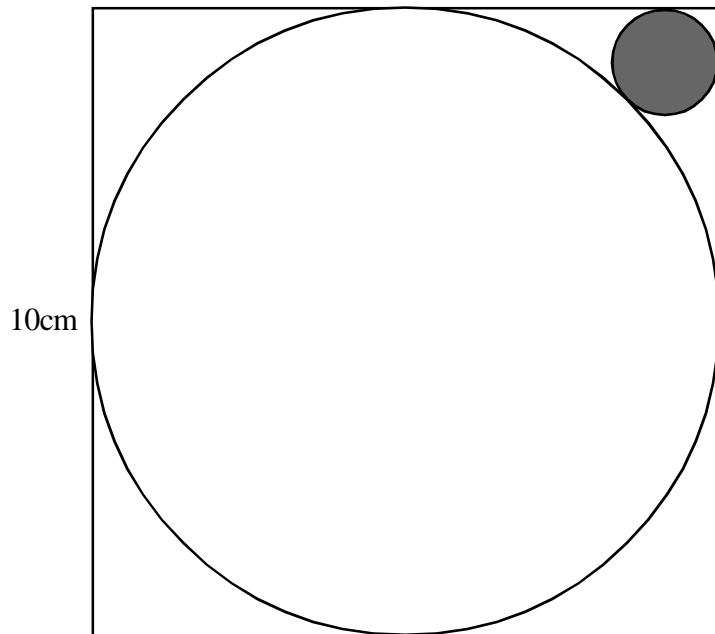
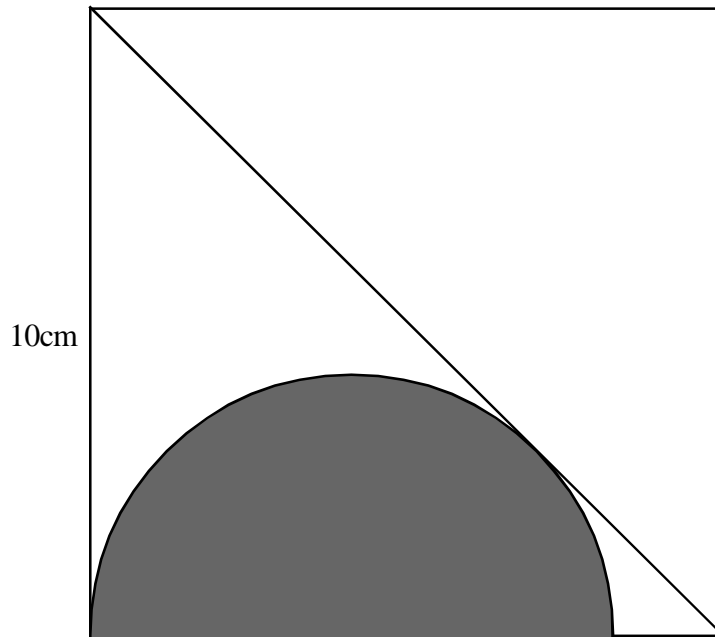
7. _____



8. _____

Geometry

Challenge set:
Find the shaded areas in terms of Pi, using radical notation.

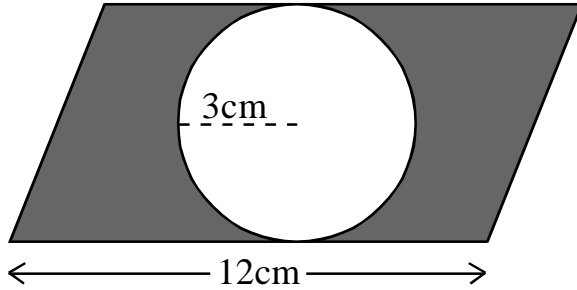


Review

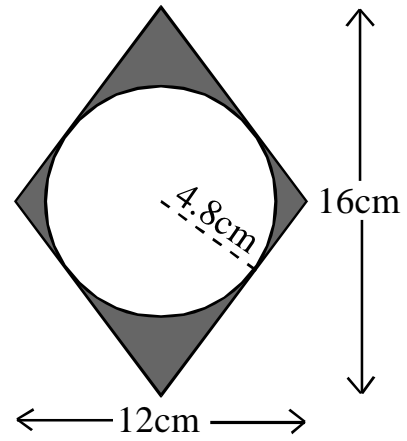
Geometry 8.6

Determine the area of each shaded region below:
Express in terms of pi and/or rounded to the tenth.

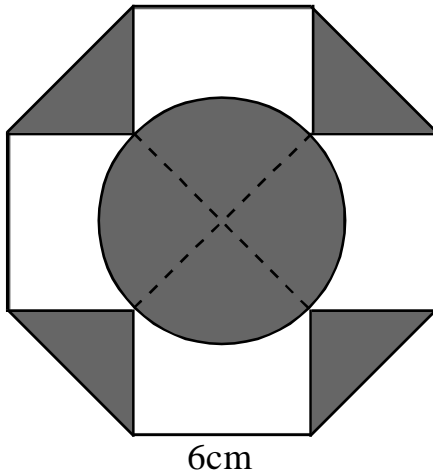
1. (parallelogram)



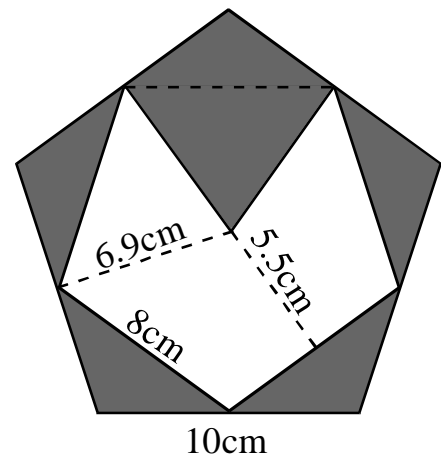
2.



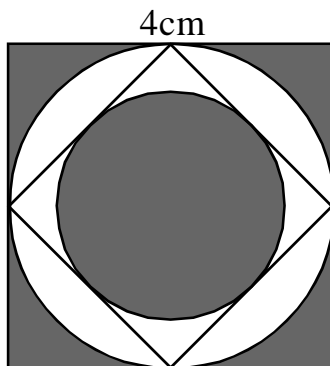
3. (octagon is regular)



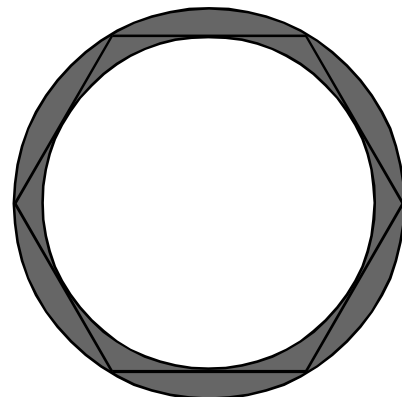
4. (pentagon is regular)



5. (circles and squares)



6. (hexagon has side length 4)



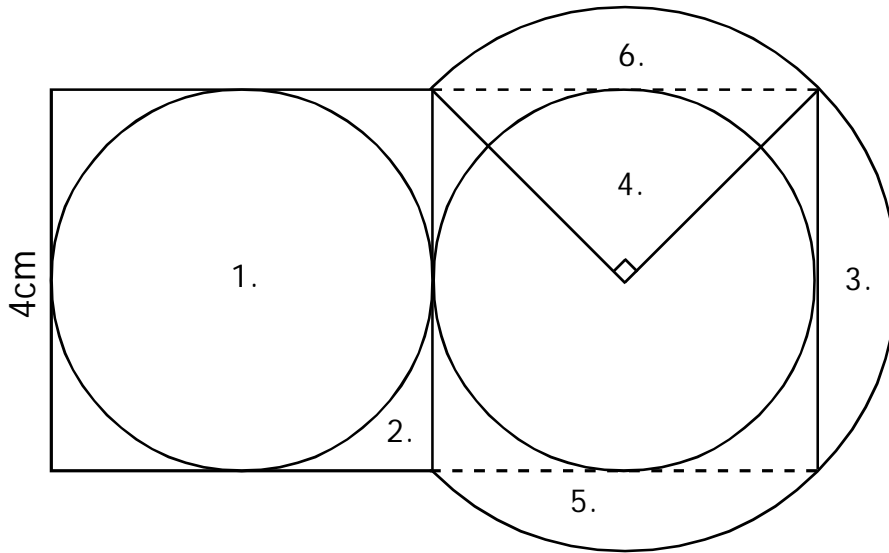
Practice

Geometry 8.6

Find the area of each numbered figure below.

Find the area of each. You may need to use the Pythagorean Theorem.

Answers should be rounded to the hundredth or in terms of pi.



1. (circle) _____

2. (???) _____

3. (segment) _____

4. (sector) _____

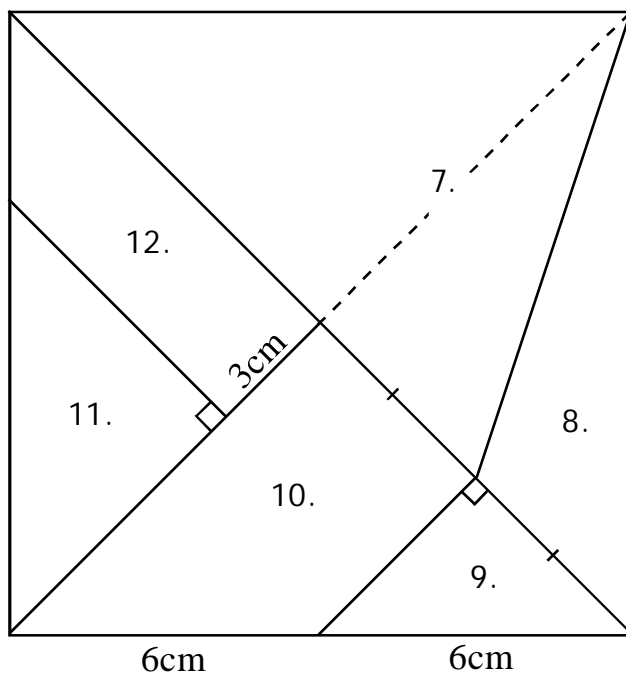
5. (???) _____

6. (partial annulus) _____

Find the area of each numbered figure below.

The figure below is a square. You may need to use the Pythagorean Theorem.

Round to hundredth (or leave answers in radical form).



7. (triangle) _____

8. (triangle) _____

9. (triangle) _____

10. (trapezoid) _____

11. (triangle) _____

12. (trapezoid) _____

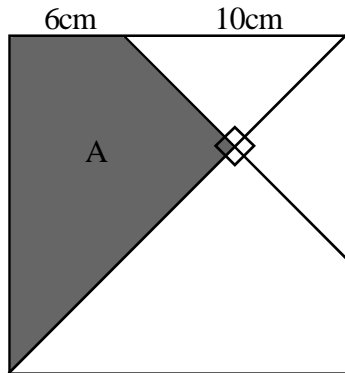
Geometric Probability

Geometry 8.6

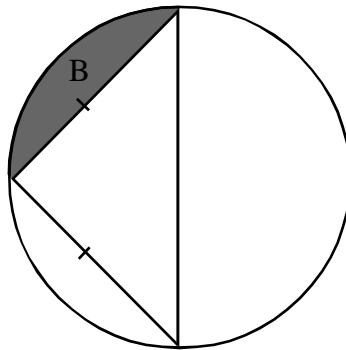
Review:

Find the labeled area in each diagram.
Round decimal answers to the hundredth.

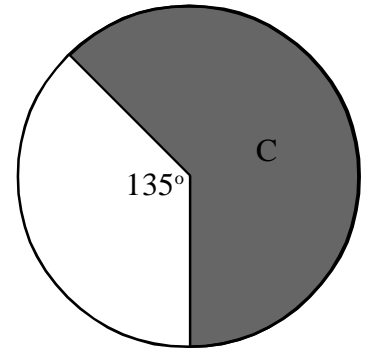
1. (figure is a square)



2. (radius=2cm)



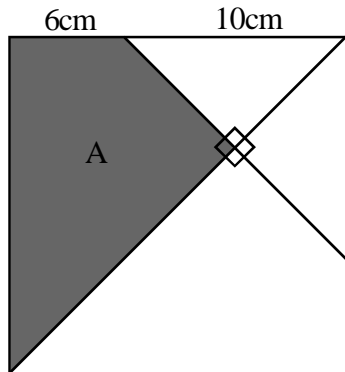
3. (radius=4cm)



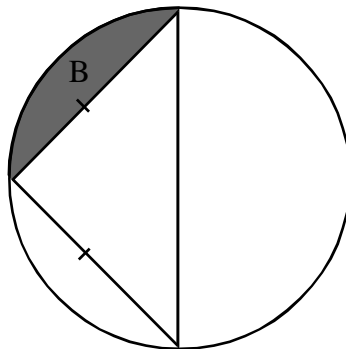
Geometric Probability:

Use your answers from above to determine **WHAT PERCENT** of the figure is shaded. Round percents to the tenth.

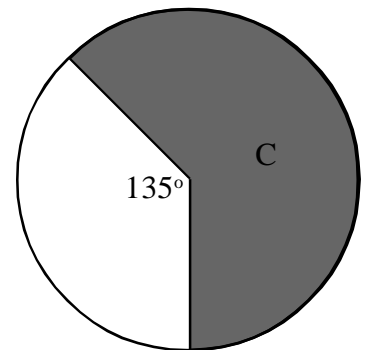
1. (figure is a square)



2. (radius=2cm)



3. (radius=4cm)



Geometric Probability:

Involves finding area ratios, usually represented as percents.

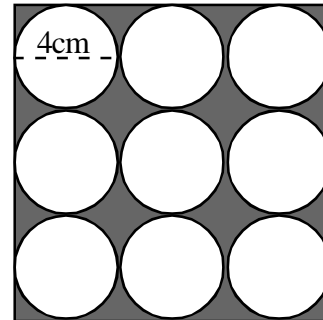
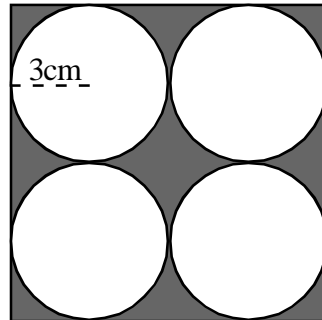
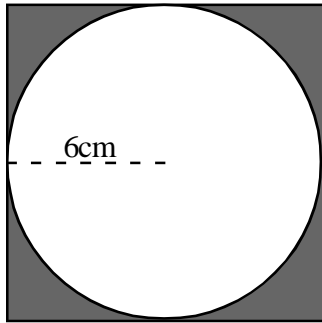
For example:

Concentric circles have radii of 3 and 4 inches. If a random point is selected within the larger circle, what is the probability that it will be within the smaller circle as well? Express your answer as a fraction and as a percent.

Geometric Probability

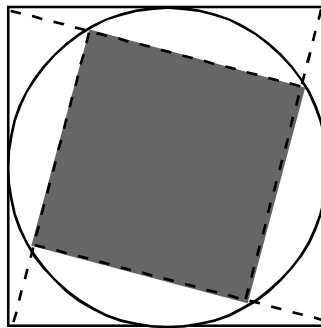
Geometry 8.6

Practice: Find the percent of each shaded area below (to the tenth):



Practice:

What percent is shaded?
(The circle is inscribed in the large square and circumscribed about the large square).

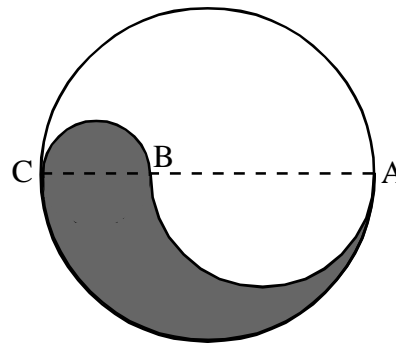


Practice:

In the diagram, $AB=40\text{cm}$, $BC=20\text{cm}$.

What is the area of the shaded region?
(in terms of π)

What percent of the circle is shaded?

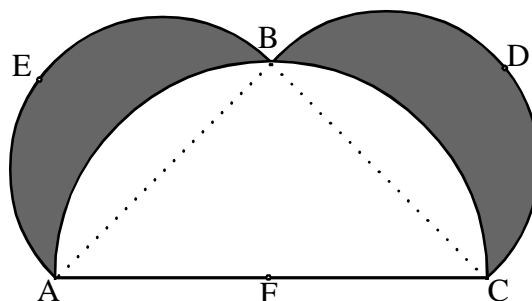


Practice:

Congruent semicircles AEB and BDC overlap semicircle ABC.
The radius of the large semicircle is 1.

What is the area of the shaded region?

What percent is shaded?



Geometric Probability

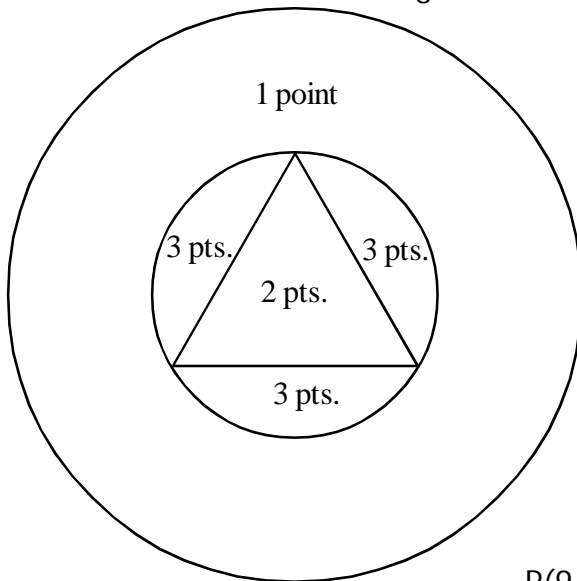
Geometry 8.6

A common problem on EOC testing involves throwing darts at an imaginary dartboard, covered with a variety of geometric shapes.

1:

A carnival game involves throwing darts at the board below. You are just good enough to hit the board every time, but your aim is not good enough to control where you hit (you hit a random point on the board every time). What is the probability that you will hit each of the areas below?

Small circle radius: 4cm Large radius: 10cm Triangle sides: $4\sqrt{3}$ cm



Give answers as percents rounded to the tenth.

P(1 point) _____ or _____%

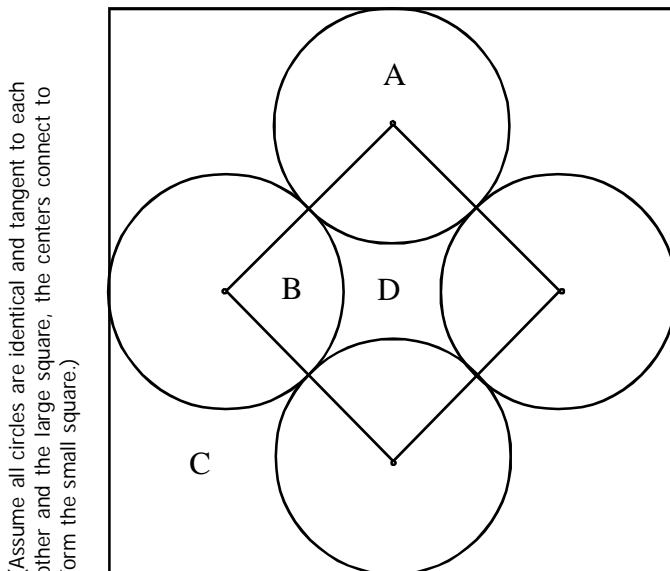
P(2 points) _____ or _____%

P(3 points) _____ or _____%

P(9 points on three darts) _____ or _____%

Practice:

Determine the probability of hitting each region with one random dart:



(Assume all circles are identical and tangent to each other and the large square, the centers connect to form the small square.)

Circle Radii = 2in

You must use the Pythagorean Theorem to find the sides of the big square.

Give answers as fractions **and** percents rounded to the tenth.

P(A) _____ or _____%

P(B) _____ or _____%

P(C) _____ or _____%

P(D) _____ or _____%

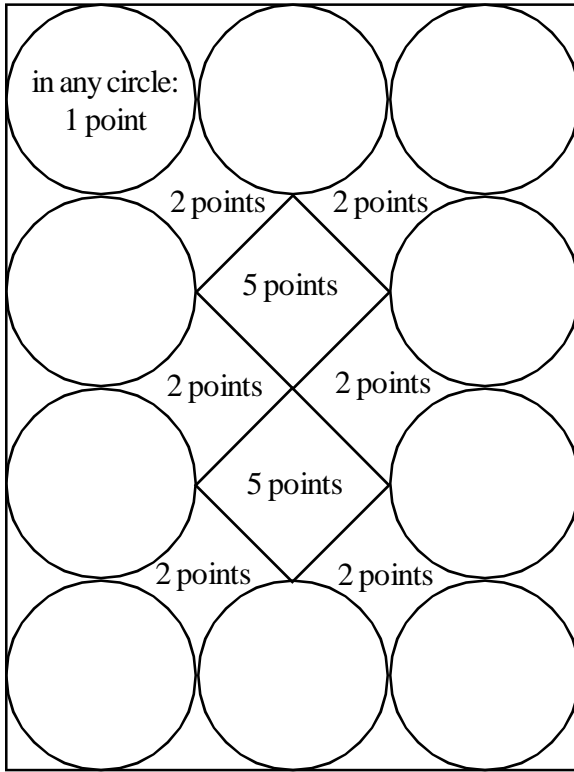
P(B or D) _____ or _____%

Geometry 8.6

Geometric Probability

Practice:

Determine the probability of hitting each region with one random dart:



Circle Radii = 4in

Give answers as fractions **and** percents rounded to the tenth.

P(1 point) _____ or _____%

P(2 points) _____ or _____%

P(5 points) _____ or _____%

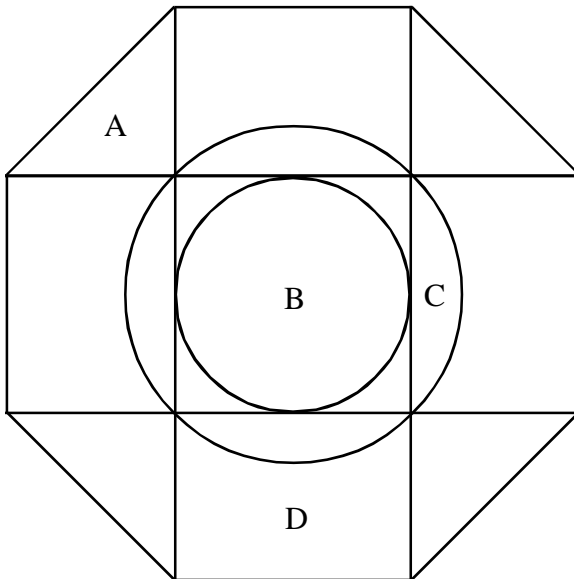
P(0 points) _____ or _____%

Add all four answers = _____%

Practice:

Determine the probability of hitting each region with one random dart:

The octagon is regular.



Octagon Perimeter: 16m

Give answers as fractions **and** percents rounded to the tenth.

P(A) _____ or _____%

P(B) _____ or _____%

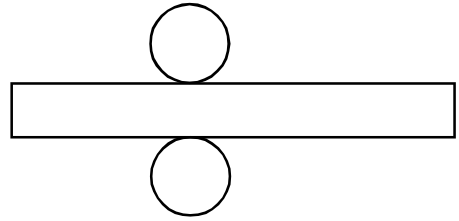
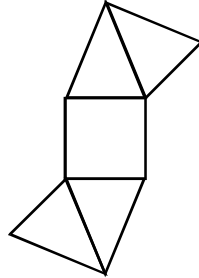
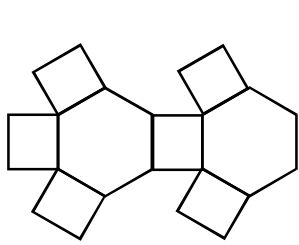
P(C) _____ or _____%

P(D) _____ or _____%

Surface Area

Geometry 8.7

Surface Area is the sum of the areas of all faces which enclose a solid.
It is useful to be able to recognize some of the nets which represent solid figures.



What solid does each figure above represent? Draw a sketch of each below.

Formulas:

Sketch each figure below and write a formula for its surface area:

Rectangular Prism

(box)

Square Pyramid

(like in Egypt)

Cylinder

(soda can)

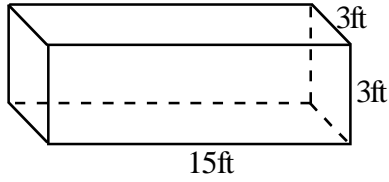
Cone

(sugar cone not a waffle cone)

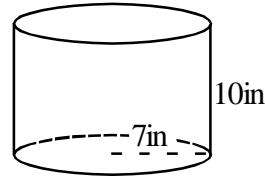
Surface Area

Geometry 8.7

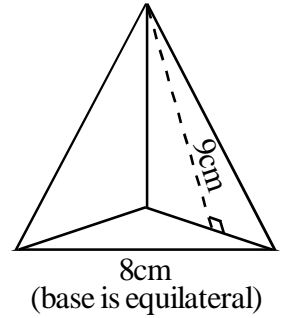
Determine the surface area of each solid below:
Round all answers to the hundredth.



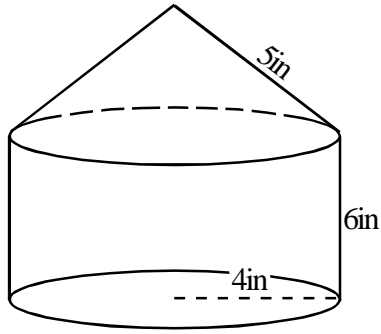
1. $A = \underline{\hspace{2cm}}$



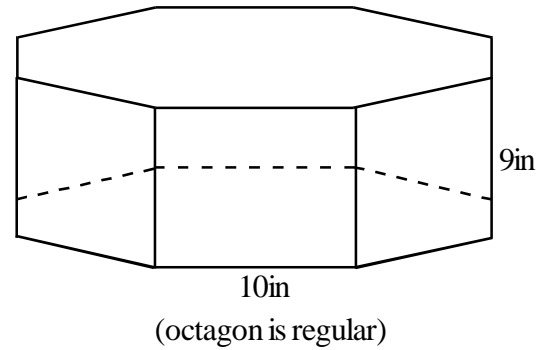
2. $A = \underline{\hspace{2cm}}$



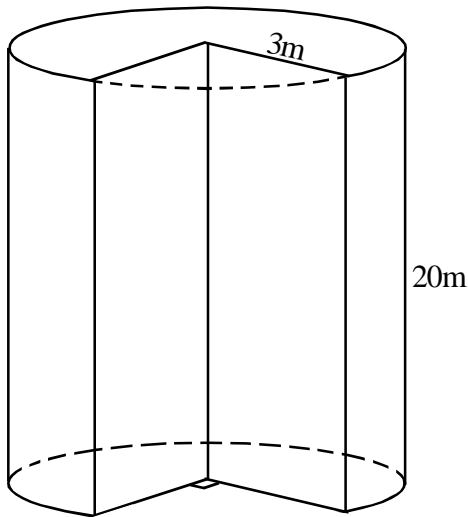
3. $A = \underline{\hspace{2cm}}$



4. $A = \underline{\hspace{2cm}}$

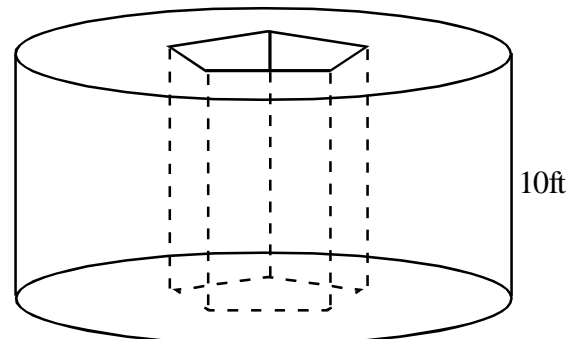


5. $A = \underline{\hspace{2cm}}$



6. $A = \underline{\hspace{2cm}}$

Circle radius: 12ft
Pentagon sides: 3ft
Pentagon apothem: 2ft



(Don't forget the surfaces 'inside' the hole.)

7. $A = \underline{\hspace{2cm}}$

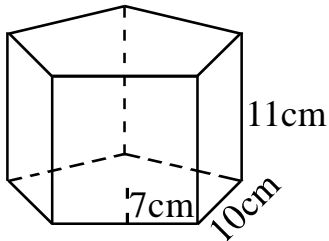
Surface Area

Geometry 8.7

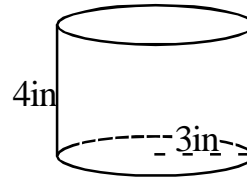
Surface areas will be generally combinations of the following types.

Find the area of each. Leave answers in radical form and/or in terms of Pi.

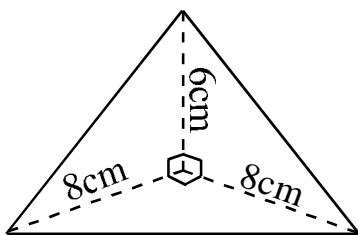
Prism



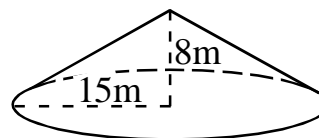
Cylinder



Pyramid



Cone

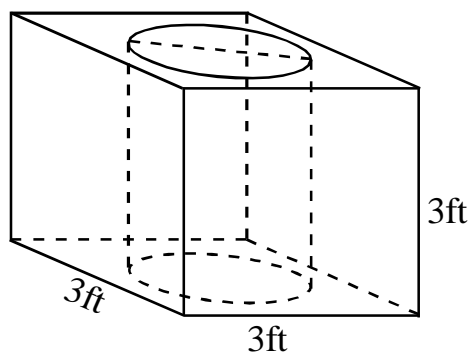


Combinations:

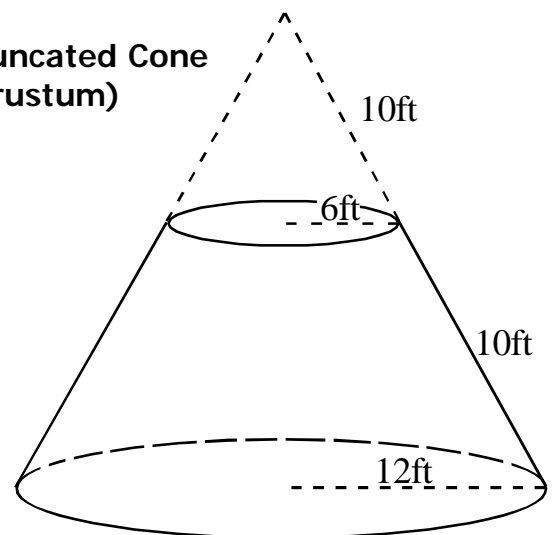
Find the surface area of each combination below.

Leave answers in radical form and/or in terms of Pi.

1. Prism with a Cylindrical Hole (diameter = 2ft)



2. Truncated Cone (Frustum)

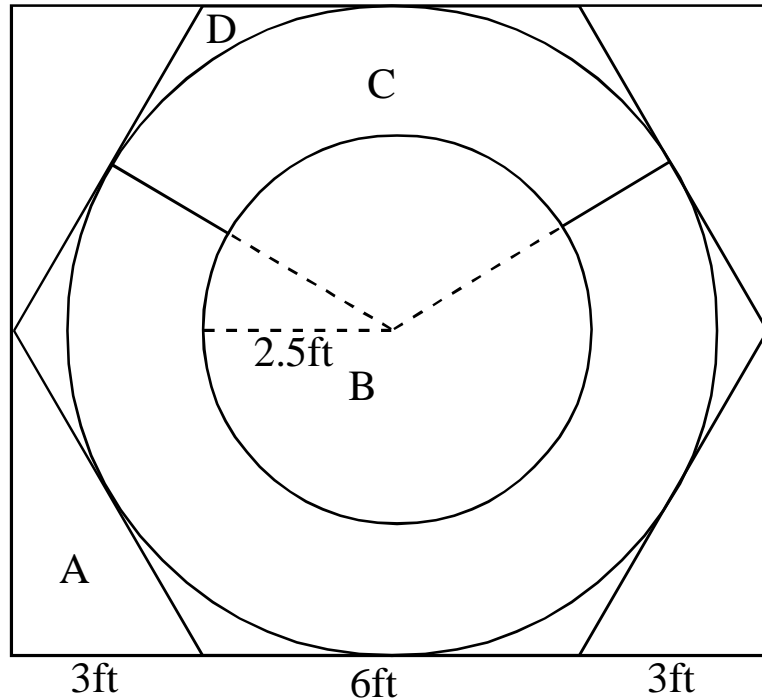


Test Review

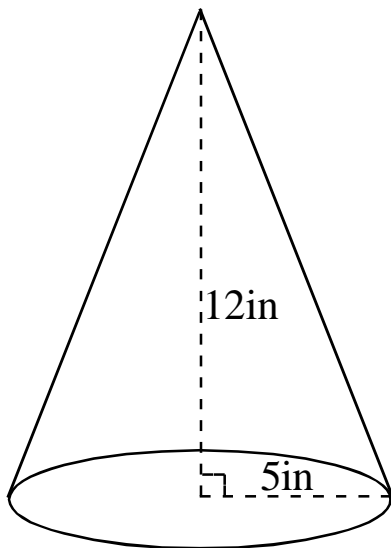
Geometry 8.7

Determine the geometric probability of striking each labeled region in the diagrams below:

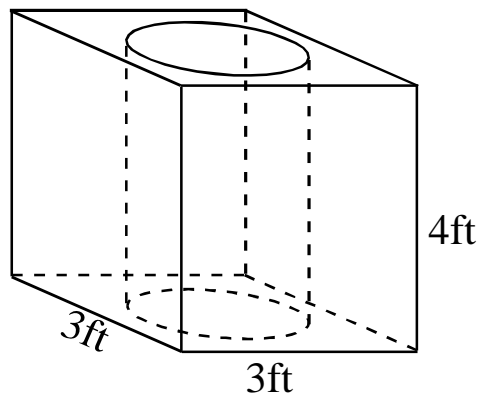
Answers should be rounded to the tenth of a percent.



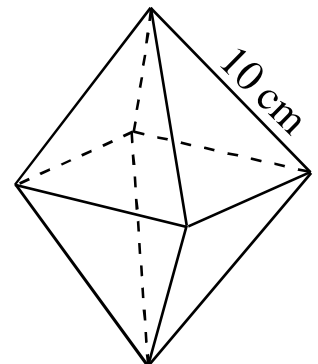
Determine the surface area or each figure below:



circle
diameter
= 3 ft



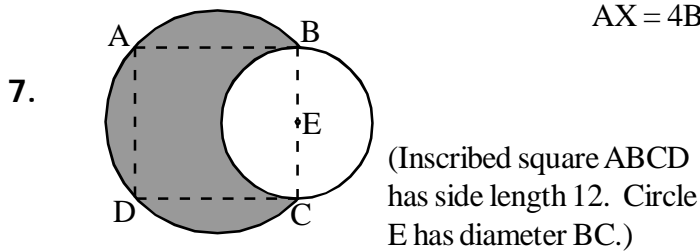
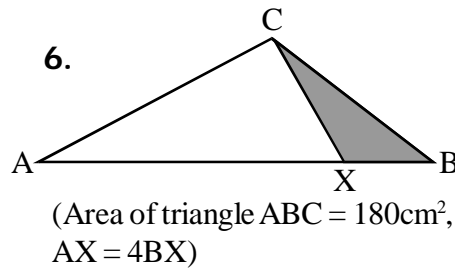
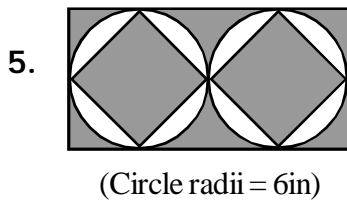
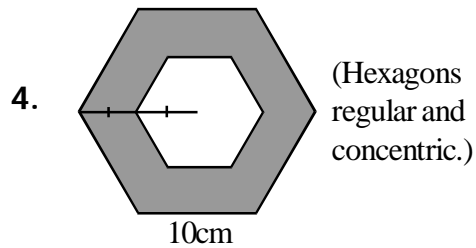
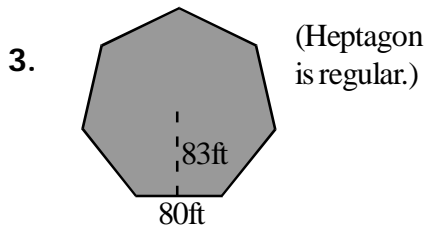
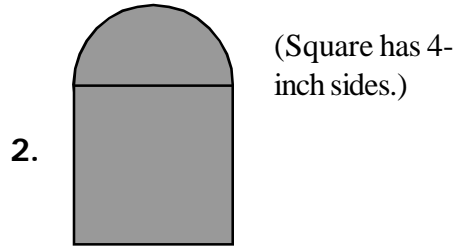
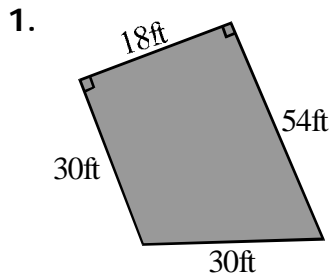
regular octahedron
(8 sides, all equilateral triangles)



Practice Test

Geometry

Determine the shaded area of each figure below. Leave answers in terms of pi and in simplest radical form where applicable.



1. _____

2. _____

3. _____

4. _____

5. _____

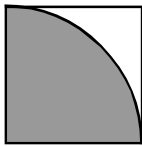
6. _____

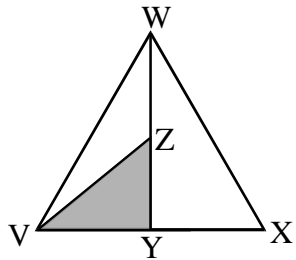
7. _____

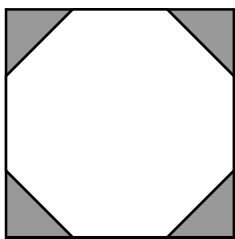
Practice Test

Geometry

What is the geometric probability of striking each shaded region below?
Round to the tenth where necessary.

8.  (Quarter-circle inscribed in a square.)

9.  (VY = XY and YZ = WZ)

10.  (Regular octagon inscribed in a square.)

8. _____
9. _____
10. _____

Find the surface area of each.

- 11-12. 11. Base = _____
12. Total = _____

- 13-14. 13. Top + Bottom = _____
14. Total = _____

