

From Counting to Probability

Probability

If you can count well, most probability problems are EASY!

Probability is simply the ratio of favorable outcomes to total outcomes.

Example:

What is the probability of rolling a 6 with a pair of standard dice?

There are five ways to roll a 6: (1,5)(2,4)(3,3)(4,2), and (5,1). There are $6^2=36$ possible outcomes when a pair of dice are rolled.

Therefore, the probability of rolling a 6 is $\frac{5}{36}$.

Try the following:

1. A positive integer less than 100 is randomly selected. What is the probability that the integer is odd?
2. There are four green blocks and four red blocks in a bag. Two blocks are selected at random. What is the probability that they are both red?
3. All of the arrangements of the letters in the word ALGEBRA are written on a list, and one of the arrangements is selected at random. What is the probability that the selected arrangement contains a double-A?

Before we go any farther, we must discuss two important terms:

Two events are **independent** if the result of the first has no effect on the second.

If two events are **dependent**, then the result of the first effects the second.

In problem number two above, we can consider the selection of two blocks as dependent events: selecting the first, then the second block. To find the probability of two events, multiply the probability of the first by the probability of the second.

Examples: There are four green blocks and four red blocks in a bag.

1. What is the probability of selecting a red block, keeping it, then selecting another red block?
2. What is the probability of selecting a green block, replacing it, then selecting a red block?

Dependent and Independent

Probability

Many probability problems involve blind selection of objects, they are easy to write and familiar. If objects are selected and replaced, the selections are independent events. Objects selected '**without replacement**' are dependent events.

Examples: Using a standard deck of cards:

52 cards in four suits, 13 in each suit: A 2 3 4 5 6 7 8 9 10 J Q K.

1. What is the probability of selecting a jack from a shuffled deck of cards?
2. What is the probability of selecting a heart then a club (with replacement)?
3. What is the probability of drawing two aces (without replacement)?
4. In a shuffled deck, what is the probability that the top four cards are all of the same suit?

Practice: Using a standard deck of cards:

52 cards in four suits, 13 in each suit: A 2 3 4 5 6 7 8 9 10 J Q K.

1. What is the probability of selecting a heart from a shuffled deck of cards?
2. What is the probability of selecting two cards from different suits (with replacement)?
3. You are dealt two card from a shuffled deck. What is the probability of getting two face cards?
4. What is the probability of selecting an Ace from the deck, keeping it, then selecting a spade? (There are multiple cases to consider: remember, the ace could be a spade.)

Would dice rolls and coin tosses be considered dependent or independent events?

Example:

A fair coin is tossed 100 times, landing 59 times on tails, and 41 times on heads. What is the probability that the next flip will be tails?

This can lead to some unsettling probabilities:

1. A standard six-sided die is thrown four times. What is the probability that all four rolls show a six?
2. A standard six-sided die is thrown three times, each time showing a 6. What is the probability that the next roll will also be a 6?

Assuming the coins and dice in problems are fair, the probability of flipping heads will always be $1/2$, and the probability of rolling a 6 will always be $1/6$, independent of any previous throws.

Compound Probability

Probability

Solve each:

1. You roll a pair of dice, one is red and the other is green. What is the probability of rolling a 5 on the red die and an even number on the green one?
1. _____
2. For a lottery drawing, balls numbered with the digits 1-9 are placed in three bins and one ball is selected from each bin in order. What is the probability that all three digits drawn will be odd?
2. _____
3. You hold the Q in a game of scrabble, and need a U. There are 7 letters left in the bag, but only one U. If you select four letters at random, what is the probability that you will get the U?
hint: What is the probability that you will NOT get a U?
3. _____
4. Six students stand in a line. What is the probability that, from left to right, they are standing in order from oldest to youngest?
4. _____
5. For a randomly selected phone number, what is the probability that the last three digits are the same?
5. _____
6. What is the probability of rolling the numbers 1 through 6 in order from least to greatest with six rolls of a standard die?
6. _____
7. What is the probability of rolling each of the numbers 1 through 6 in any order with six rolls of a standard die?
7. _____
8. In six rolls of a standard die, what is the probability that the same number will be rolled exactly five times?
(This requires some insight and counting techniques.)
8. _____

Counting and Probability

Probability

Sometimes it is not enough to understand how to compute compound probability. Often, there are multiple ways in which a successful outcome can occur. In this case, it is often useful to compute the probability of a series of events occurring one way, then multiply by the number of ways it can happen.

This makes more sense in an example:

Three fair dice are rolled. What is the probability that exactly two of the rolls show a 1?

Solution 1:

Compute the probability of rolling a 1-1-X (where X is not a 1):

$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{5}{216}$, but you can also roll 1-X-1 or X-1-1 with the same probability.

$$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot 3 = \frac{15}{216} = \frac{5}{72}$$

Solution 2:

Compute the number of favorable outcomes and divide by the number of possible outcomes (6^3).

You are filling three slots ____ ____ _____. There are three ways to place the ones, and the final slot can be filled with any one of 5 digits.

$$\frac{3 \cdot 5}{6^3} = \frac{15}{216} = \frac{5}{72}$$

Practice:

1. A coin is flipped 7 times. What is the probability that exactly three of the flips are tails?
2. Marlon randomly selects three stones from a bag containing two red stones, two black stones, and two white stones. What is the probability that Marlon selects two red stones and one black stone?
3. Lisa shakes her piggy bank until five coins fall out. She knows that there are five nickels and five quarters in her piggy bank. What is the probability that the sum of the change that has fallen out is \$0.85? (Assume nickels and quarters fall out with equal probability.)

Counting and Probability

Probability

Solve each:

1. A homeroom has 7 boys and 5 girls. The homeroom randomly selects two representatives to student council. What is the probability that the homeroom elects one boy and one girl?

1. _____

2. Four students each take a free-throw. If each student has a $\frac{2}{5}$ chance of making his/her free-throw, what is the probability that exactly three students make their shot?

2. _____

3. The school cafeteria has chocolate chip cookies and oatmeal cookies for dessert, and every student gets a cookie. If students choose chocolate chip over oatmeal 2 to 1, what is the probability that exactly 3 of the next 5 students will select chocolate chip cookies?

3. _____

4. In the problem above, what is the probability that exactly 4 of the next 5 students will select chocolate chip cookies?

4. _____

5. Twenty-seven students each flip a fair coin. What is the probability that there are more heads showing than tails?

5. _____

6. Three students are randomly selected from a group of twelve, including twins Billy and Adam. What is the probability that Adam is selected, but Billy is not?

6. _____

Challenge: Alice and Ben take turns rolling a standard six-sided die. The first person to roll a six wins. If Alice goes first, what is the probability that she will win the game?

C. _____

Casework and Probability

Probability

Sometimes probability problems involve solving problems for multiple cases.

Consider the following:

A fair coin is tossed five times. What is the probability of flipping at least two heads?

Solution 1: Calculating 2,3,4, or 5 heads, there are $2^5=32$ possible outcomes, with $5C5=1$ plus $5C4=5$ plus $5C3=10$ plus $5C2=10$ or $1+5+10+10$ ways out of $32 = 26/32$ or $13/16$.

Solution 2: How could you use the entries in Pascal's Triangle to solve this problem?

Solution 3: Complementary counting: flipping 0 heads can happen one way, flipping one heads can happen 5 ways. This means $6/32$ times you will not flip at least two heads, leaving $26/32$ or $13/16$ ways to flip at least two heads.

Solution 4: You may recognize that half of the time you will flip more heads than tails, or $16/32$. This means we only need to find out how often we will flip two heads: $5C2=10$, which means $26/32$ or $13/16$ you will flip 2 or more heads.

Practice:

1. A pair of dice is rolled. What is the probability that the product of the two numbers rolled is greater than or equal to 20?
2. Jack flips a penny twice, and Kelly flips a nickel three times. What is the probability that each flips the same number of heads?
3. A bag contains seven red marbles and three green marbles. Opal selects three marbles from the bag at random and without replacement. What is the probability that she selects more red than green marbles?

Challenge:

In a set of five index cards, three are blue on both sides, one is white on both sides, and one has a blue side and a white side. A randomly selected card is placed on a table and shows a blue face. What is the probability that the other face is also blue?

Casework and Probability

Probability

Solve each:

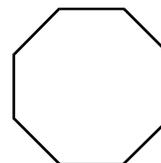
1. The numbers 1 through 9 are placed in a bag. Juan selects three numbers from the bag at random and without replacement. What is the probability that the three numbers Juan picks are consecutive integers?

1. _____

2. Marty has a drawer containing three white socks, four blue socks, and five black socks. If Marty randomly selects two socks from the drawer, what is the probability that the socks will be the same color?

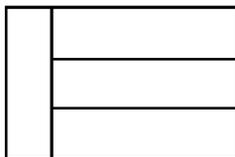
2. _____

3. Four of the vertices of a regular octagon are selected at random and connected to form a quadrilateral. What is the probability that the quadrilateral will be a rectangle?



3. _____

4. An organization is choosing colors for the stripes on its flag. The flag has three horizontal stripes and a vertical stripe as shown. The color of each of the four stripes is selected at random from the colors red, white, and black. What is the probability that no two stripes of the same color share an edge?

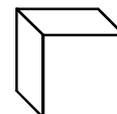


4. _____

5. Three of the vertices of a regular heptagon are connected to form a triangle. What is the probability that the triangle formed is isosceles?

5. _____

6. Each of the four faces of a cube is painted either white or black with equal probability. What is the probability that no two black faces share an edge?



6. _____

Review (Includes Counting)

Probability

Sticks and Stones:

1. How many ways can seven soccer balls be divided among three coaches for practice?

1. _____

2. How many ways can eight reindeer be divided among three sleighs if at least one reindeer must pull each sleigh?

2. _____

3. How many ways can six slices of pepperoni pizza and three slices of cheese pizza be divided among five students if one of the students is a vegetarian? (Hard, skip this one and come back to it.)

3. _____

Pascal's Triangle

1. Add: $\binom{5}{5} + \binom{6}{5} + \binom{7}{5} + \binom{8}{5}$

1. _____

2. What is the probability that in seven flips of a coin, there are four, five, or six tails?

2. _____

3. $\binom{25}{20} + \binom{25}{21} = \binom{n}{r}$. Find $n + r$.

3. _____

Review (Includes Counting)

Probability

Counting and Compound Probability:

1. A bag contains 10 black blocks and four white blocks. What is the probability of selecting three black blocks from the bag without replacement?

1. _____

2. Fifteen students (six boys and nine girls) are randomly assigned into three groups of five. What is the probability that one of the groups has all girls?

2. _____

3. You are dealt three cards from a standard deck. What is the probability of being dealt three face cards, all from different suits?

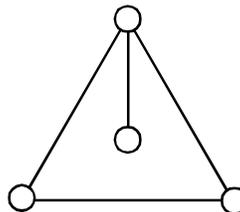
3. _____

Casework and Probability:

1. Three of the vertices of a cube are connected to form a triangle. What is the probability that all three vertices are on the same face of the cube?

1. _____

2. Each of the four dots below is randomly assigned a color: blue, purple, or navy. What is the probability that no two dots that are the same color will be connected by a line segment?



2. _____

3. Three digits are selected at random and without replacement from a bag containing each of the digits 0-9. What is the probability that all three digits share a common factor greater than 1?

3. _____

Practice Quiz

Probability

Solve:

1. Add: $\binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8}$

1. _____

2. You are dealt two cards from a standard deck. What is the probability that they are both red numbered cards? (2-10 of either hearts or diamonds)

2. _____

3. A bag contains only red blocks, white blocks, and black blocks. The probability of selecting a red block is $\frac{1}{5}$, and the probability of selecting a white block is $\frac{3}{8}$. What is the fewest number of black blocks that could be in the bag?

3. _____

4. Martha is making four giant cookies, each shaped like a different animal. She has ten chocolate kisses to distribute among the four cookies. How many ways can she distribute the ten kisses, if each cookie must receive at least one chocolate?

4. _____

5. Ms. Smith tells her students that there are at least 3 "true" answers on an eight-question true/false quiz. How many possible combinations of answers are there on the quiz?

5. _____

6. Jeremy has found a way to flip a coin so that it comes up tails twice as often as heads. What is the probability of Jeremy flipping more tails than heads in three coin flips using this method?

6. _____

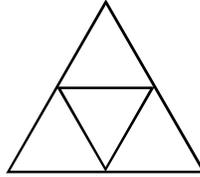
7. Oleg rolls a standard die three times. What is the probability that the third roll is greater than the sum of the first two rolls?

7. _____

Practice Quiz

Probability

8. Each of the triangles in the diagram below is painted either yellow, orange, or red with equal probability. What is the probability that no two triangles that are the same color share an edge?



8. _____

9. Hank can hit the bull's eye with a bow-and-arrow one in three times. If Hank fires nine arrows, what is the probability that exactly three of them will land in the bull's eye? Express your answer as a fraction in simplest form or rounded to the tenth of a percent.

9. _____

10. What is the probability of being dealt a face card then a spade from a standard deck of 52 cards?

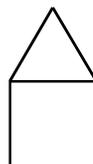
10. _____

Counting and Probability Quiz

Probability

Solve:

- What is the sum of the terms in the tenth row of Pascal's Triangle, which begins: 1 10 45 ...
1. _____
- Ryan has made three snowmen, and is using coals to make the buttons on the front of the snowmen. If there are seven coals, how many ways can they be distributed among the three snowmen?
2. _____
- There are two dozen (24) marbles in a bag, each is either clear, blue, green, or white. If the probability of randomly selecting a blue marble is $\frac{1}{3}$, and the probability of selecting a red marble is $\frac{1}{4}$, and you are twice as likely to draw a clear marble than a white marble, how many white marbles are in the bag?
3. _____
- You are dealt three cards from a standard deck of 52 cards. What is the probability that exactly two cards are from the same suit?
4. _____
- At Ashwin's birthday party, five tables are needed to seat all of the invited guests. There are a dozen red balloons for decorating the tables at the party. You want the birthday boy's table to get at least five balloons. How many ways are there to arrange the balloons among the five tables?
5. _____
- Three distinct digits are selected from the digits 2 through 8. What is the probability that the sum of the three digits is a prime number?
6. _____
- Each of the six segments in the drawing below is randomly painted either red or blue. What is the probability that there will be a red square or triangle in the diagram?
7. _____



Counting and Probability Quiz

Probability

8. The first ten letters of the alphabet are randomly placed into two equal piles of five letters each. What is the probability that the letters A, B, and C are all in the same pile of letters?

8. _____

9. On average, it is cloudy for 12 of the 28 days in February. What is the probability that during the second week of February there will be exactly one day that is cloudy? Express your answer to the nearest tenth of a percent.

9. _____

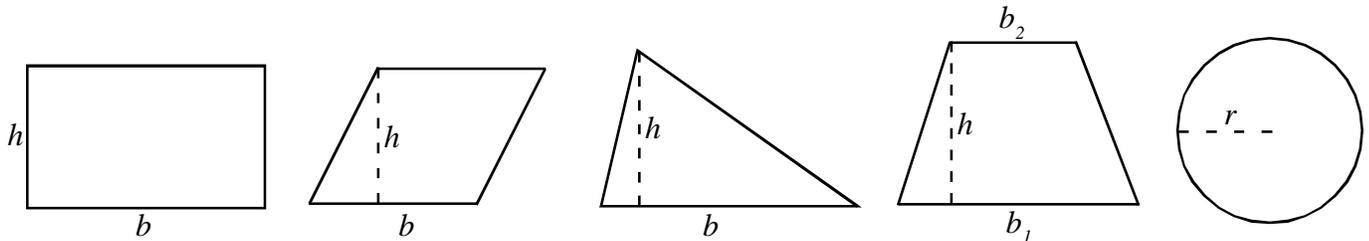
10. Henry's Ice Cream shop offers eight toppings for its ice cream sundaes. You order the deluxe special, which allows you to pick five or more of the available toppings. How many combinations of toppings can you choose from?

10. _____

Geometric Probability

Probability

Many, many random darts have been thrown at dart boards in Math Land. When calculating geometric probabilities, it is useful to have a general knowledge of some basic area formulas:



Most of the time, geometric probability problems are simply solved by finding a fractional area:

Example:

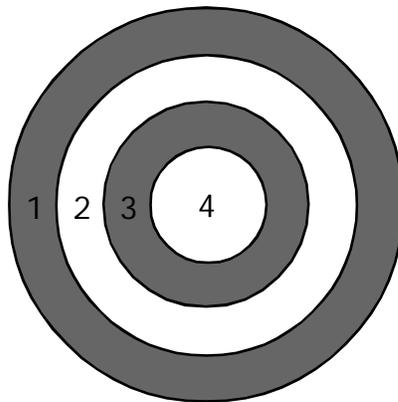
A random dart is thrown at the board below, which is made up of concentric circles of radius 3, 6, 9, and 12cm. What is the probability of the dart hitting each numbered ring of the dart board?

$P(1 \text{ point}) =$

$P(2 \text{ points}) =$

$P(3 \text{ points}) =$

$P(4 \text{ points}) =$



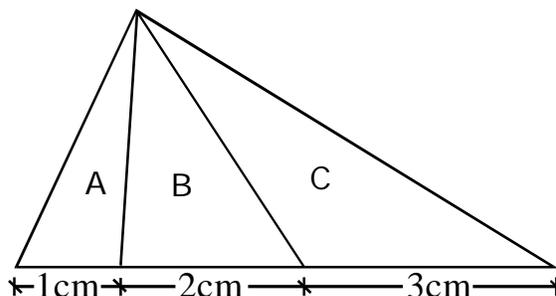
Example:

A random dart is thrown at the board below, which is made up of triangles of base 1cm, 2cm, and 3cm. What is the probability of the dart hitting each lettered triangle of the dart board?

$P(A) =$

$P(B) =$

$P(C) =$



Geometric Probability

Probability

Dartboard problems can become more complicated:

Example:

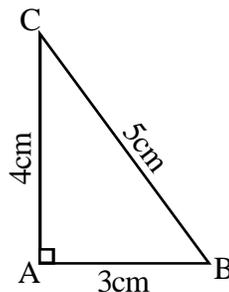
What is the probability of scoring *more* than four points with two random darts thrown at the board below?



Other geometric probability problems involving area:

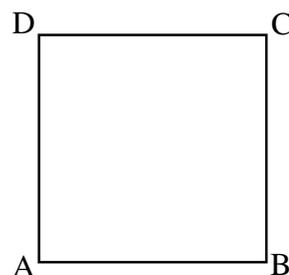
Example:

Point X is selected in the interior of 3-4-5 right triangle ABC. What is the probability that the area of triangle ABX will be greater than 3cm^2 ?



Example:

Point X is selected at random inside square ABCD. What is the probability that the area of triangle $ABX > BCX > ADX$?

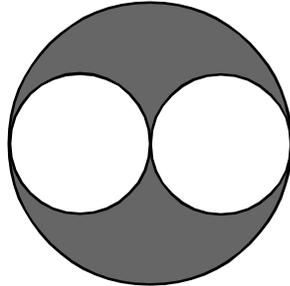


Geometric Probability

Probability

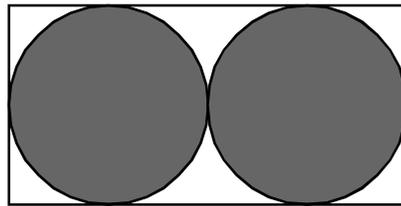
Solve each:

1. A random point is selected within the large circle below. What is the probability that the point will be in the shaded region, outside of the two congruent smaller circles?



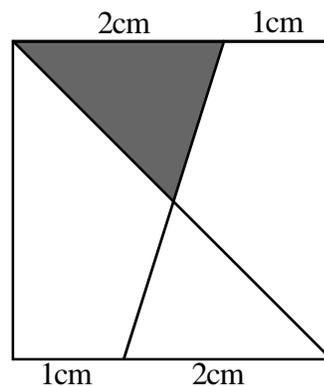
1. _____

2. A random point is selected within the rectangle below. What is the probability that the point will lie in the shaded region within one of the two congruent circles? Express your answer in terms of π .



2. _____

3. What is the probability that a random point selected within the square below will be in the shaded area?



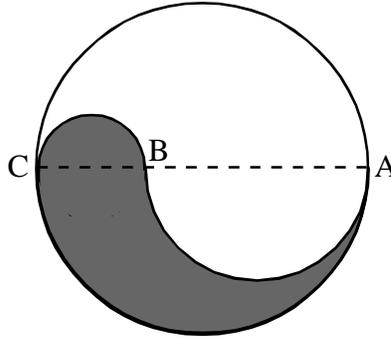
3. _____

Geometric Probability

Probability

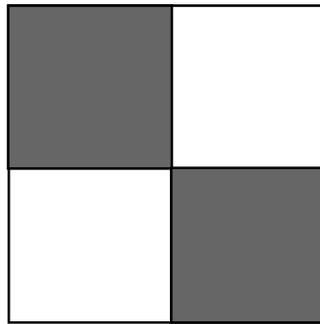
Solve each:

4. A random point is selected within the large circle below. What is the probability that it lands in the circle below. If $AB = 6$ and $BC = 3$, what is the probability that the point is within the shaded area?



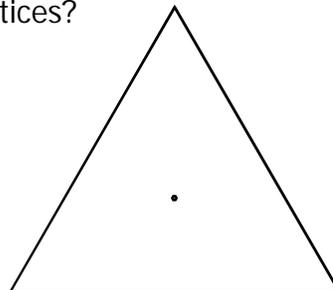
4. _____

5. A half-dollar coin with a 1cm radius is tossed onto a table with 16cm sides. The coin *may* land with an edge hanging off of the table (as long as it doesn't fall off). What is the probability that the coin lands so that it is touching (touching includes being completely within) one of the shaded squares?



5. _____

6. A point is selected at random on the perimeter of an equilateral triangle. What is the probability that it will be closer to the center of the triangle than to any of the three vertices?



6. _____

Geometric Probability

Probability

Sometimes Geometric Probability problems are not as obvious. Graphing can be a useful tool when probability problems seem to have countless possible outcomes.

Example:

A positive real number $0 < x < 10$ is selected at random and added to a second randomly selected number $0 < y < 5$. What is the probability that the result will be greater than 5 but less than 10?

Example:

A positive real number $0 < x < 1$ is selected at random. A second number $0 < y < 2$ is selected at random. What is the probability that $x < y$?

Example:

The bus comes to a stop near your house every day at a random time between 6:45 and 6:50. You arrive at the bus stop at a random time between 6:40 and 6:45 every day and wait until the bus comes. What percent of the time will you wait less than 4 minutes for the bus to arrive?

Practice:

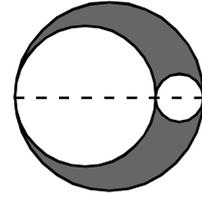
1. Two real numbers a and b are selected at random between -5 and 5 . What is the probability that $a + b > a - b$?
2. A positive real number $0 < x < 1$ is selected at random. A second number $0 < y < 1$ is selected at random. What is the probability that $x + 2y > 2$?
3. Driving to grandma's for Thanksgiving, uncle Paul plans to arrive between 6 and 7pm, while aunt Sue plans to arrive between 6:30 and 7:30 pm. What is the probability that they will arrive within 15 minutes of each other if each arrives randomly within the expected time frame?
4. You know that there is a lunar eclipse tonight, but you have forgotten what specific time it will begin. You only recall that the eclipse lasts an hour and will begin some time between 11pm and 5am. You set your alarm for a random time between 11pm and 4am, and go out for an hour to try to catch it. What is the probability that you will witness at least a portion of the eclipse?

Geometric Probability

Probability

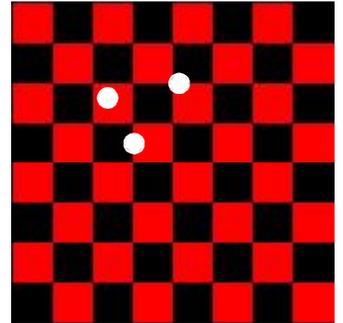
Solve each:

1. What is the probability that a point selected within the circle below will be within the shaded region if the radius of the large circle is 4cm and the radius of the small circle is 1cm? Express your answer as a percent.



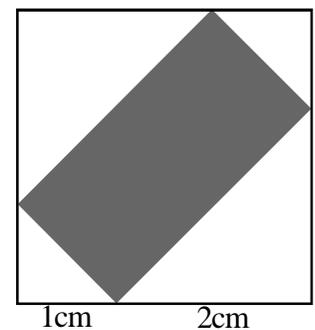
1. _____

2. In a carnival game, round discs are tossed onto a checkered grid made up of 4-inch black and red squares. If the disc lands completely within a red square, you win a prize. What is the probability of winning a prize if the discs have a 1-inch radius? (Assume that the discs are always tossed in the interior of the checkered board, not near the edges.)



2. _____

3. A point is selected at random within the square below. What is the probability that it lands within the shaded region?



3. _____

Geometric Probability

Probability

Solve each:

4. What is the probability that two numbers selected at random between 1 and 4 have a sum greater than 5?

4. _____

5. One side of a triangle is 5cm long. Two numbers are selected between 0 and 10. What is the probability that the two numbers can be the other two sides of the triangle (for example, 9 and 3 could not be, but 9 and 5 could be. Make sure you understand why before beginning).

5. _____

6. Pamela is meeting a friend for lunch. She arrives at a random time between 12 and 12:30 and waits 15 minutes before leaving (if Corey does not arrive in this time). Corey is less patient. He arrives between 12 and 12:30 but only waits for 5 minutes (if Pamela does not arrive). What is the probability that the two will eat together?

6. _____

Expected Value

Probability

Expected value is easiest to think of as the average outcome. The expected value is not necessarily the value that is most likely to occur.

Example:

When you roll a standard six-faced die, you win whatever number you roll in dollars. What are the expected winnings on a single roll of the die?

Example:

A more complicated game involves rolling a pair of dice. Winnings are calculated as follows: You win whatever amount you roll unless you roll doubles (1-1, 2-2, etc.), in which case you lose triple the amount rolled. What are the expected winnings on a single roll of the die?

Example:

A change machine is broken. If you put in a penny, it spits out a coin at random: either a penny, nickel, dime, or quarter, each with equal probability. If you put in a dollar's worth of pennies, how much money can you expect to get back in change?

Practice:

You and a friend play a game of chance which involves flipping two quarters each. Find your expected winnings for each turn given the set rules below.

1. If three or four coins show the same (heads or tails), you win them all. If there are two of each (2 heads and 2 tails) your friend keeps all the coins. What is your expected value for each turn?
2. If three or four coins show the same (heads or tails), your friend keeps the coins in the majority (ex. HHHT means your friend keeps the three H coins). If there are two of each (2 heads and 2 tails) you keep all the coins. What is your expected value for each turn?
3. Your friend flips his coins first. If you can match his flip (HH, HT, or TT) you keep the coins. If you cannot, he keeps the coins. What is your expected value for each turn?
4. Your friend flips three coins to your two. If he flips more heads, he keeps the coins, but if you flip as many or more heads, you keep the coins. What is your expected value for each turn?

Expected Value

Probability

Solve each:

1. There are three red marbles and seven blue marbles in a bag. You select two marbles at random. If they are the same color you win a dollar, and if they are different you lose a dollar. What is the expected value for the selection of one marble from the bag?

1. _____

2. The faces on a six-faced die are numbered 1, 1, 2, 3, 5, and x . If the expected value of a roll on the die is 3, what is the value of x ?

2. _____

3. Eight prizes are randomly assigned to briefcases with values of \$0, \$2, \$10, \$50, \$100, \$250, \$500, and \$1000. You select a briefcase at random. What is the expected value of your briefcase?

3. _____

4. A red die and a green die are rolled. If the value of the green die is greater than the value of the red die, you win the difference. Otherwise, you roll again until you win. What is the expected value of a turn.

4. _____

5. There are tokens in a bag worth \$1 and \$5. When you select a token at random, the expected value is \$1.20. What is the fewest number of tokens that could be in the bag?

5. _____

Expected Value

Probability

Solve each:

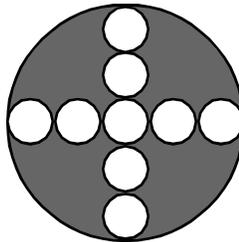
6. A multiple choice test has 20 questions, and each question has five answer choices. Correct answers are worth 5 points, blank answers are worth 1 point, and incorrect answers are worth 0 points.

- a. What is the expected value of a random guess?
- b. What is the expected value of a guess if you can eliminate one of the five answer choices?

a. _____

b. _____

7. A dart strikes a random point on the board below. What is the expected value for a single dart if the small circles are worth 2 points and the remaining area is worth 1 point?



7. _____

8. What is the expected value for the sum of the digits on a digital clock?



8. _____

Mean, Median, and Mode

Probability

Solve:

1. What is the mean of the first ten positive odd integers?
1. _____
2. The average for Caleb's first 7 tests this year is 91. What score will he need to average on his next two tests if he wants to improve his test average to 93?
2. _____
3. The mean of a set of seven positive integers is 7, the median is 8, and the distinct mode is 9. What is the greatest possible range for the set of integers?
3. _____
4. At track practice, Caeman runs 15 laps. The mean of Caeman's first three lap times was 71 seconds, and the mean of his next five times was 69 seconds. If his average time for the entire practice was 68 seconds, what was Caeman's average lap time for his last seven laps?
4. _____
5. Harold scores an 80 on his tenth quiz, increasing his quiz average by 2 points. What is his new quiz average?
5. _____
6. A set of eight distinct positive integers has a mean of 9 and a median of 5. An additional integer is added to the set, increasing the mean to 10. What is the smallest possible range for the new set of integers?
6. _____

Mean, Median, and Mode

Probability

Solve:

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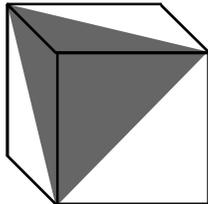
Quiz Review

Probability

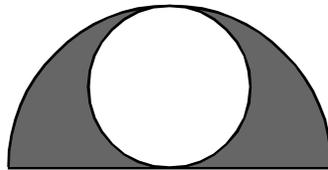
Geometric Probability

1. What is the probability that a random point selected on the surface of the cube will be within one of the three grey triangular shaped regions (none of the hidden faces are grey)?

1. _____

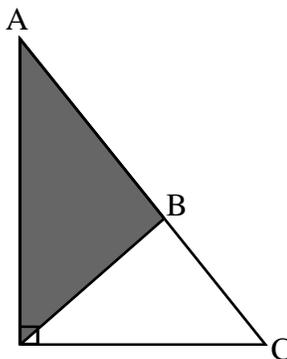


2. A point is selected within the semicircle below. What is the probability that it will be outside the small circle?



2. _____

3. A point is randomly selected inside the large right triangle below. What is the probability that it will fall within the shaded region if $AB = 3$ and $BC = 2$.



3. _____

4. Two numbers are selected at random. If $1 < a < 5$ and $2 < b < 6$, what is the probability that $a/b > 0.5$? Express your answer as a common fraction in simplest form.

4. _____

Quiz Review

Probability

Expected Value

5. A standard die has faces numbered 1, 2, 4, 8, 16, and 32. What is the expected value for a single roll of the die?

5. _____

6. A newspaper has five sheets, folded in half, and each side of each sheet is numbered (1-20, starting with the front of the newspaper with page 1). The paper is opened to a random page in the middle, and the sum of the two page numbers is taken. What is the expected value of the sum.

6. _____

7. The mean of a set of eight distinct integers is 18.5. What is the minimum range for the set?

7. _____

8. The mean for a set of nine positive integers is 13. Two consecutive integers are added to the set, and the mean is increased by 3. What are the two integers?

8. _____

9. If Tanya scores an 80 on her next test, her test average will be an 83. If she scores a 92 on her next test, her average will be an 85. How many tests has Tanya taken so far?

9. _____

Practice Quiz

Probability

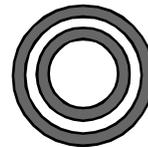
Solve:

1. What is the probability that a random point selected within the rectangle below will be within the shaded triangular region?



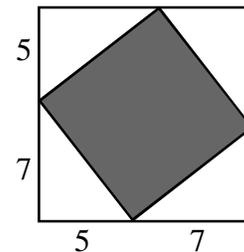
1. _____

2. Concentric circles below have radii of 3, 4, 5, and 6cm. What is the probability that a random point selected within the large circle is in one of the shaded regions?



2. _____

3. What is the probability that a random point selected within the large square below will be within the smaller shaded square?



3. _____

4. A penny, a nickel, a dime, and a quarter are all flipped. You get to keep all the coins that land heads side up. What is your expected value?

4. _____

5. Four numbers are added to the set of integers below, increasing the mean, median, and mode each by 1. What is the largest of the the integers?

3 4 5 5 8

5. _____

Practice Quiz

Probability

Solve:

6. Two positive numbers less than 10 are selected at random. What is the probability that the sum of the selected numbers is less than 6?

6. _____

7. The expiration date on a credit card is given as two digits for the month and two digits for the year. What is the expected value for the sum of the digits in the expiration date for a credit card that expires between 01/10 and 01/15?

7. _____

8. Austin has calculated that if he scores a 95 on his next three tests, it will improve his test average to 93. If Austin has taken six tests so far this year, what is his current average?

8. _____

9. Point X is selected at random within square ABCD of side length 3. What is the probability that quadrilateral ABXD has an area greater than 4 square units?

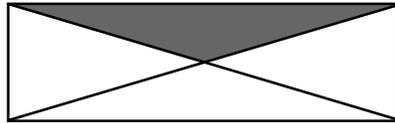
9. _____

10. Three white, four red, and five blue blocks are placed in a bag and selected at random, two at a time, and without replacement. If the blocks are the same color, you win \$3 if they are either both red or both blue, and \$6 if they are both white. What is the expected value for a draw of two blocks?

10. _____

Quiz: Geometric Probability, Expected Value, Stats.**Probability****Solve:**

1. What is the probability that a random point selected within the rectangle below will be within the shaded triangular region?

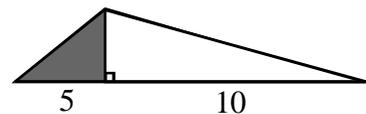


1. _____

2. How many distinct sets of three positive integers have a mean of 6, a median of 7, and no mode?

2. _____

3. What is the probability that a random point selected within the large triangle will be within the smaller shaded triangle?



3. _____

4. Tiles numbered with each of the digits 1 through 5 are placed in a bag, so that there are the same number of tiles for each number as the number on the tile (for example, there are 4 tiles with the number four). What is the expected value for the draw of a single tile? Express your answer as a common fraction in simplest form.

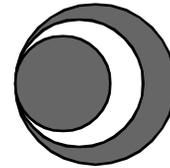
4. _____

5. If your average for the first three quizzes in this class is a 90, how many of the ten questions on this quiz will you need to get right to have an 'A' average (92.5 or above). Remember, you get 60 points plus 5 points per correct answer.

5. _____

Quiz: Geometric Probability, Expected Value, Stats.**Probability****Solve:**

6. The circles shown have radii of 3, 4, and 5cm. What is the probability that a random point selected within the large circle is in one of the shaded regions?



6. _____

7. A row or column of three numbers is selected on a standard calculator with the numbers 1 through 9 arranged as shown. What is the expected value for the sum of the three selected numbers.

7	8	9
4	5	6
1	2	3

7. _____

8. To improve his quiz average by 1 point, Carter needs to score a 94 on his next quiz. To improve his average by 2 points, he needs to score a 101. on his next quiz. What is Carter's current quiz average?

8. _____

9. In rectangle ABCD, $AB = 1$ and $BC = 2$. Point X is selected at random within the rectangle. What is the probability that the area of triangle ABX is more than twice the area of triangle BCX?

9. _____

10. A bag contains eighteen \$5 tokens. There are also an equal number of \$1 and \$2 tokens in the bag. The expected value for the draw of a token at random is \$2.25. If there are twelve \$5 tokens in the bag, how many tokens are in the bag altogether?

10. _____