Solutions: Challenge Set 1

1. $(2 + 4 + \ldots + 100) - (1 + 3 + \ldots + 99) = (2-1) + (4-3) + \ldots + (100-99) = 50(1) = \mathbf{50}$.

2. There are 5 faces, 9 edges, and 6 vertices. $5 + 9 + 6 = \mathbf{20}$.

3. The ratio of the time it takes Seth to solve a problem to the amount of time Greg takes is 7:5, so for every 5 problems that Seth solves, Greg solves 7. Seth will solve 15 for every 21 that Greg solves, so Greg will be expected to solve **6** more than Greg if they attempt 36 problems together.

4. In three days, you ordered a total of two of each item and spent $2 + $2.50 + $3 = $7.50. To buy one of each item would cost half that: **$3.75**.

5. Convert $x + y = xy + 13$ into $x - xy + y = 13$, then $x(1-y) - 1(1-y) = 13 - 1$, so $(x-1)(1-y)=12$. We are looking for integer solutions, and 12 has 6 positive factors. The value of $(x-1)$ can be any of those 6 factors, and can also be the negative value of any one of those 6 factors: (13, 0) (7,-1) (5,-2) (4,-3) (3,-5) (2,-11) (0,13) (-1,7) (-2,5) (-3,4) (-5,3) (-11,2) are the **12** ordered pairs. (Look-up Simon's Favorite Factoring Trick for more information on the factoring above).

6. There are 10 faces which do not overlap, each with an area of 4cm^2. The two overlapping faces are each missing $\frac{1}{4}$ of their areas, so each has a remaining surface of 3cm^2. $10(4) + 2(3) = \mathbf{46}\text{cm}^2$.

7. Draw altitude BD (with D on BC). Triangle ABD is a 30-60-90 right triangle with BD = 3cm and AD = $3\sqrt{3}$. This makes CD = 6cm, and by the Pythagorean Theorem we find that $AC = \sqrt{63} = \mathbf{3\sqrt{7}}\text{cm}$.

8. If 32 teams play 16 games, we get a total of $(32 \times 16)/2 = 256$ winners. If we divide these wins amongst teams who have 9 wins, there are $256/9 = \mathbf{28}$ (with a remainder of 4) teams who could have 9 wins.

9. Assume there are 100 coins … there would be 95 pennies and 5 other coins worth a total of $0.95. This is only possible with 2 dimes and 3 quarters. 2/100 = **2%** of the coins are dimes.

10. Method 1: There are 9! Ways to arrange the 9 letters in FACETIOUS. In each arrangement, consider rearranging only the vowels. There are 5! ways to arrange the vowels and only one way is alphabetical. This gives us 9!/5! = 3,024 arrangements with the vowels listed alphabetically.

    Method 2: Consider filling nine blanks with the letters. Place the four consonants F, C, T, and S first. Fill the remaining blanks with the vowels in the correct order. There are 9 places to put the F, 8 to place the C, 7 to place the T, and 6 to place the S for $9 \times 8 \times 7 \times 6 = \mathbf{3{,}024}$ ways to arrange the consonants and only one way to place the vowels after the consonants are all placed.

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