

Proportions and Percents

Percent means **per hundred**.

Think of some words that contain the word cent:

Fractions can be used to represent **ratios**.

For example:

$\frac{3}{5}$ can be used to represent "three out of five".

One of the easiest ways to change a **ratio to a percent** is by using a proportion.

Example: Express three out of five as a percent:

You may have learned the following:

$$\boxed{\frac{\textit{is}}{\textit{of}} = \frac{\%}{100}} \quad \text{Is over of equals percent over 100.}$$

This is the useful for very simple problems involving percents:

Examples: Use the Percent Proportion to Solve:

1. What percent is 12 of 40?
2. 6 is 30% of what number?
3. What is 20% of 45?

I prefer you use the following variation of **the percent proportion**:

$$\boxed{\frac{\textit{Part}}{\textit{Whole}} = \frac{\%}{100}} \quad \text{The part over the whole equals percent over 100.}$$

Examples: Use the Percent Proportion to Solve:

1. There are 60 words altogether on the vocabulary list. Jennifer knows 65% of them. How many of the words does she know?
2. Lewis took 15% of the candy in the bag. If Lewis took 12 pieces, how much candy was there in the bag?
3. 19 of the fish are spotted. If there are 50 fish in the tank, what percent of the fish have NO spots?

Percents and Proportions

Algebra

Solve each using a proportion. Round decimal answers to the tenth.

- ____ 1. What percent is 7 of 40?
- ____ 2. What is 15% of 20?
- ____ 3. 12 is 60% of what number?
- ____ 4. Ryan made 7 out of 20 free-throw attempts. What percent is this.
- ____ 5. 90% of the chocolates sold were milk chocolate. If 80 chocolates were sold, how many were milk chocolate?
- ____ 6. What percent is 15 of 75?
- ____ 7. There are 12 girls in the class. If 20% of the students in class are girls, how many boys are in the class?
- ____ 8. Six of the numbers on the list are odd numbers. If there are 16 numbers on the list, what percent of them are odd numbers?
- ____ 9. The football team has won 35% of its games in the past 10 years. If they have lost 52 games, how many games have they won?
- ____ 10. 52% of the undergraduate students at UNC are female. If there are 6,292 female undergraduate students at UNC, how many undergraduates attend UNC all together?
- ____ 11. What percent of 90 is 15? Express your percent answer as a mixed number.
- ____ 12. Jon read 85% of the book assigned for homework. The book is 320 pages long. How many pages does Jon have left to read?
- ____ **Challenge 1.** Carrie has won 7 of her first 15 tennis matches. How many wins does she need in a row to improve her winning percent to 75%?
- ____ **Challenge 2.** John has a bag full of marbles. Fifty percent of the marbles in the bag are red, but if he adds three red marbles to the bag, 60% of the marvbles in the bag will be red. How many marbles are in John's bag?

Percent Change:

$$\frac{\text{change}}{\text{original}} = \frac{\% \text{ change}}{100}$$

Change over original value equals %/100

Examples: Use the Percent Proportion to Solve:

1. The price on a shirt went from \$15 to \$12. What was the percent off?
2. Cary has a population of about 100,000 residents, up from only 60,000 just 15 years ago. By what percent has Cary's population increased?
3. Grant grew by 15% in the past two years. If he was 60 inches tall two years ago, how tall is he now?
4. You must pay 9% sales tax on all prepared foods. If a Happy Meal costs \$3.25, what is the price after sales tax (rounded to the cent)?

More Percent Change:

$$\frac{\text{new}}{\text{original}} = \frac{\text{new \%}}{100}$$

New amount over the original equals new%/100.

This makes some problems much easier when you don't know the change. Try the previous set using this formula.

Examples: Use the Percent Proportion to Solve:

1. Best Buy decreased the cost of its best flat screen monitor by 20%, and it is now being sold for \$429.99. What was the cost before the discount (to the cent)?
2. Belle improved her fast pitch speed by 25%. If she was pitching 48mph before, what speed can she pitch now?
3. The population of Bobaloobaville increased by 8% last year. If there are 21,060 people in Bobaloobaville now, how many were there a year ago?
4. Mr. Lyons improved his mile time by 5%. Last year he could run the mile in 6 minutes. What is his mile time this year?

Percent Change

More Percent Change:

Why you need *new over original*:

The most common mistake that people make when solving percent change problems occurs on questions like this:

Ex: Brandon weighs 10% more than Phillip. If Brandon weighs 220 pounds, how much does Phillip weigh?

Many students get 198. Why!?!

Try solving these problems mentally:

1. You have \$100. You lose 10% of your money, then gain 10%. How much do you have now?
2. You have \$100. You lose 50% of your money. What percent would you need to gain to have \$100 again?

This is why we need: $\frac{\text{new}}{\text{original}} = \frac{\text{new \%}}{100}$

Examples: Use the Percent Proportion to Solve:

1. Mr. Batterson invested money in a stock that has increased in value by 44% and it is now worth \$5,760. What was the value of the original investment?
2. Deborah improved her quarter mile time by 6.6 seconds, and she now runs the quarter-mile in 59.4 seconds. What percent improvement is this?

Practice: Use the Percent Proportion to Solve:

1. Find the original price on a refrigerator if you paid \$588.49 after a 7% sales tax.
2. Sarah improved 100m dash time from 12.21 seconds to 11.48 seconds. By what percent did her time decrease? (to the nearest percent)
3. Antonio is trying to gain weight for football in high school. He weighs 140 pounds now and wants to weigh 161 pounds before tryouts. What percent of weight gain is this?
4. Anna scored a 767 on her most recent math SATs. This is 18% better than her previous score. What was her original SAT score?

Challenge:

Phillip weighs 20% less than Brandon. What percent would Phillip need to gain to weigh the same as Brandon?

Percent Change Shortcuts

Decimals and Percents

To convert a percent to a decimal, move the decimal point two places to the left.

Easy:

$25\% = \underline{\hspace{2cm}}$

$14\% = \underline{\hspace{2cm}}$

$6\% = \underline{\hspace{2cm}}$

Harder:

$25,000\% = \underline{\hspace{2cm}}$

$.014\% = \underline{\hspace{2cm}}$

$6.06\% = \underline{\hspace{2cm}}$

Try using *is over of* on the following three problems:

1. What percent of 340 is 51?
2. 51 is 15% of what number?
3. What is 15% of 340?

For #3 there is an easier way:

To find a percent of a number, convert the percent to a decimal and multiply.

Ex. What is 12% of 180?

Practice:

1. What is 25% of 190?
2. What is 10% of 34?
3. What is 3.5% of 650?
4. What is 450% of 19?

This is especially helpful in problems involving a percent increase or decrease:

Ex. Find the price after tax on the following items using the given tax. Round to the cent.

1. \$40.00 (5%)
2. \$22.50 (7.5%)
3. \$314.99 (3.9%)

Practice:

1. A volleyball is being sold for 25% off. If the original price was \$15.96, what is the sale price?
2. The cost of gas increased last week by 2%. If the original cost was \$2.50/gallon, what is the new cost?

Challenge: A textbook you need for college has been reduced in price by 30%, and you have a coupon which allows you to save an additional 25%. If tax is 5%, how much will you pay for the book which was originally \$40?

Percents and Proportions

Algebra 4.4

Solve. Use the skills we have learned so far to answer the following:

1. What number is 22% of 280? 1. _____

2. What percent is 123 of 164? 2. _____

3. What number is 12% more than 425? 3. _____

4. 20% less than a number is 76. What is the number? 4. _____

5. Tax on a \$60.95 pair of shoes is 5%.
How much will you pay after tax? 5. _____

6. Corey has 22% more money than Carla. If Corey has \$30.50, how
much money does Carla have? 6. _____

7. Sears marked up all washer/dryer prices by 8%. If the original price
on a Kenmore washer was \$350, what is the price after markup? 7. _____

8. Apples are on sale: buy four get one free. This is the same as getting
what percent discount? 8. _____

9. After spending \$8.40 on lunch Kayla now has 88% of the cash left in
her purse. How much money did she start with? 9. _____

10. Jeremy has 20% more money than Sue, who has 20% less money
than Richard. If Jeremy has \$57.60, how much does Richard
have? (hint: correct answer is a whole dollar amount) 10. _____

Percents and Proportions

Algebra 4.4

Solve. Find the cost of each item after the discount and/or tax.
Round answers to the cent.

11. **Skis:** \$248.90
Tax: 7%

Price: _____

12. **Envelopes:** \$2.95
Tax: 5%

Price: _____

13. **Sofa:** \$598.95
Tax: 10%

Price: _____

14. **Cereal:** \$4.89
Tax: 5.5%

Price: _____

15. **Computer:** \$875.55
Discount: 10%
Tax: 3%

Price: _____

16. **Dining Table:** \$185
Discount: 40%
Tax: 8%

Price: _____

17. **Television:** \$428.99
Discount: 20%
Tax: 6%

Price: _____

18. **Breakfast:** \$14.18
Tax: 4.5%
Tip: 20%

Price: _____

Practice: Dilations on the Plane

Algebra 4.4+

Graphing figures on the coordinate plane is simple.

Graph and connect each set of points below separately.

Connect each set in order and then connect the first and last points in each set.

1. $(2,3)$ $(9,3)$ $(9,7)$ $(2,7)$

What shape is this?

What quadrant is it in?

Find its area.

2. $(-3,-3)$ $(-7,-2)$ $(-8,6)$ $(-4,7)$

What shape is this?

What quadrant is it in?

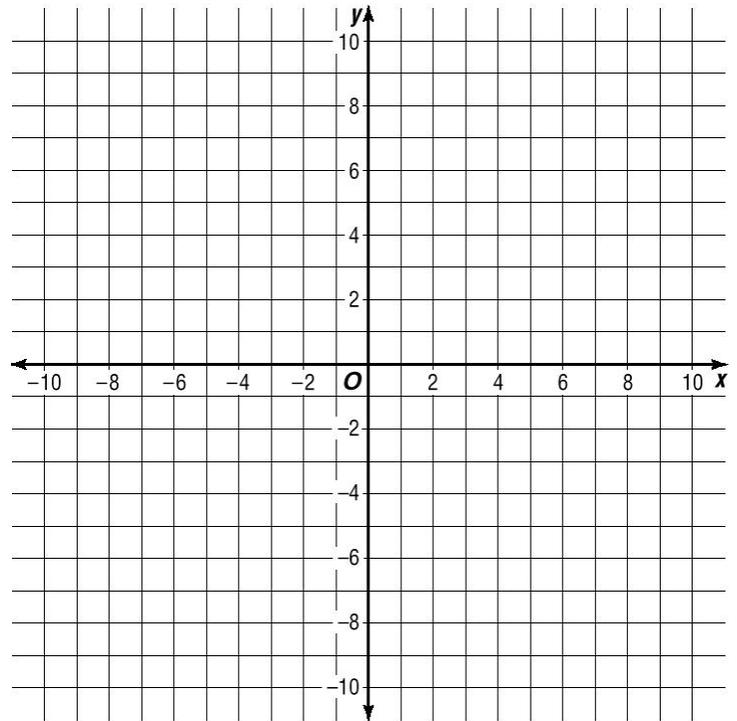
Find its area.

3. $(-7,-7)$ $(-3,-9)$ $(6,-6)$ $(2,-4)$

What shape is this?

What quadrant is it in?

Find its area.



A **dilation** is a reduction or enlargement of the original figure. To create a dilation on the coordinate plane, multiply each coordinate by a **scale factor**.

Practice:

Graph the following triangle.

- $(-1, 2)$ $(2,1)$ $3, -2)$

Find its area.

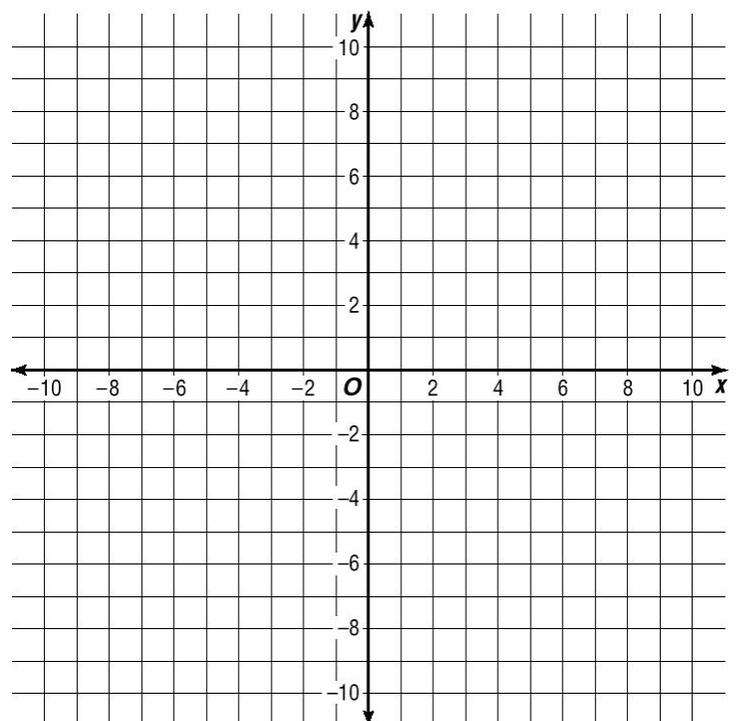
Dilate the original triangle with a scale factor of 2.

Find its area.

Dilate the original triangle with a scale factor of 2.5.

Find its area.

Predict the area of a dilation of the original with a scale factor of 5.



Practice: Dilations on the Plane

Algebra 4.4+

Plot each set of points and the dilations listed and answer the questions that follow.

1. $(0, -2)$ $(2, 2)$ $(4, -2)$

Find its area. _____

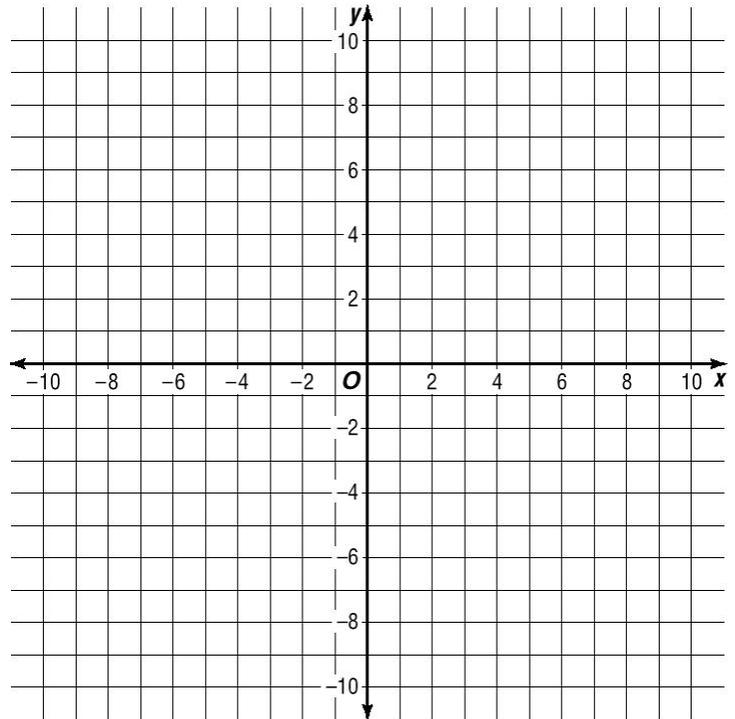
2. Graph a dilation with a scale factor of 2.

Find its area. _____

3. Graph a dilation with a scale factor of 2.5

Find its area. _____

4. Predict the area of a dilation with a scale factor of 10.



5. $(-3, 1)$ $(1, 1)$ $(2, -2)$ $(-2, -2)$

Find its area. _____

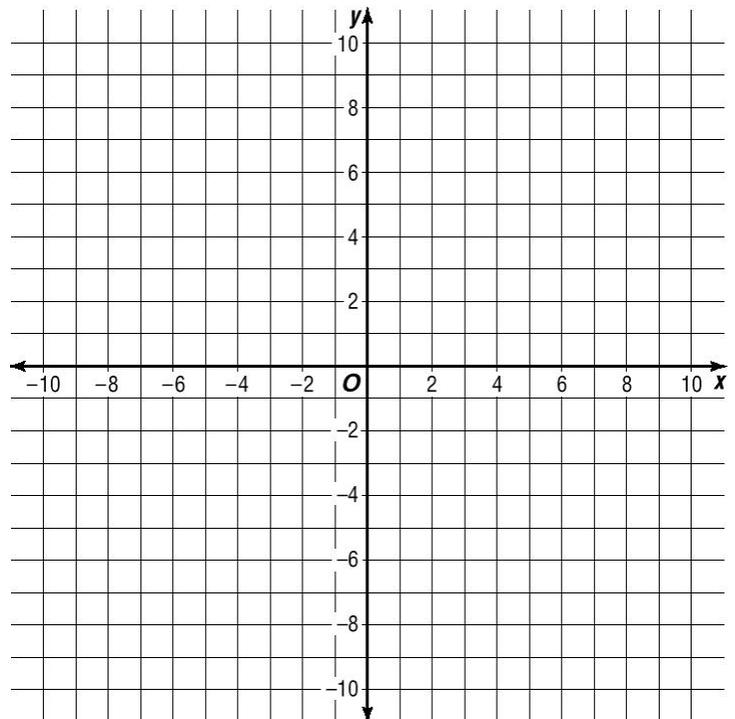
6. Graph a dilation with a scale factor of 2.

Find its area. _____

7. Graph a dilation with a scale factor of 3.

Find its area. _____

8. Predict the area of a dilation with a scale factor of 6.



Dilations on the Plane

Algebra 4.4+

Determine the scale factor used to dilate each pair.
Some answers may be fractions.

9. a to b _____

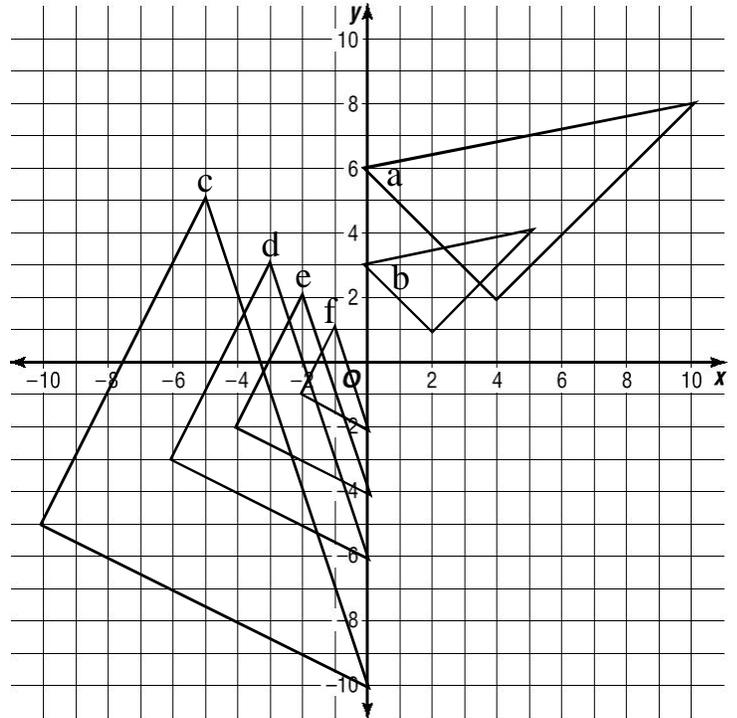
10. b to a _____

11. f to c _____

12. f to d _____

13. e to c _____

14. c to d _____



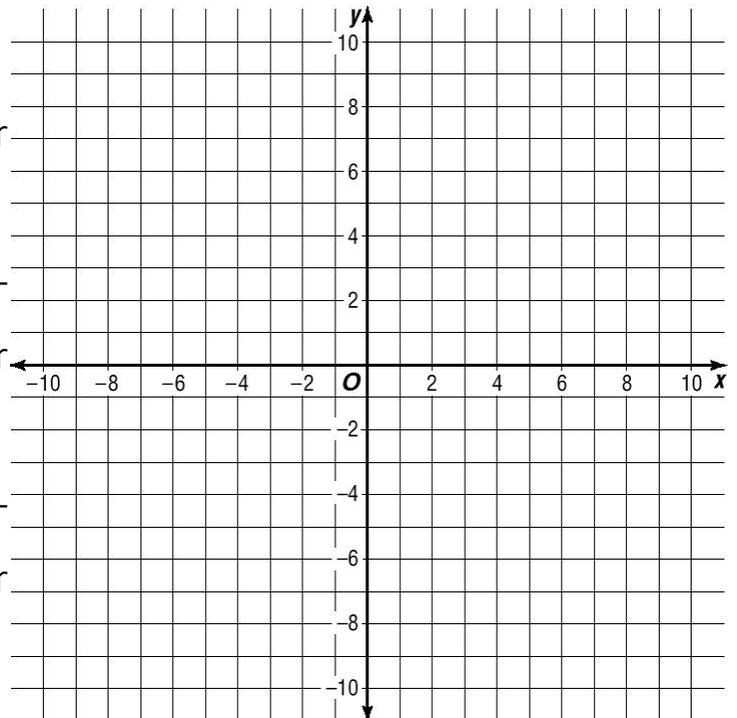
Complete the following dilations below.

15. Original:
(-4, 4) (-8, -8) (8, -4)

16. Dilation with a scale factor
of 1/4.

17. Dilation with a scale factor
of 0.75.

18. Dilation with a scale factor
of 1.2



Similar Triangles

Algebra 4.4+

Proportions and Similar Triangles:

Similar triangles are the SAME SHAPE not the same size.

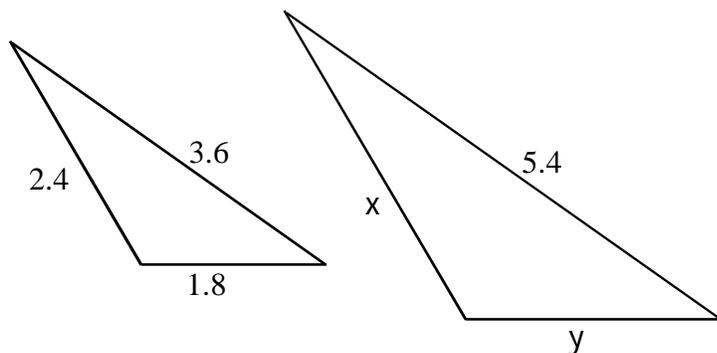
Corresponding angles are equal.

Corresponding sides are proportional.

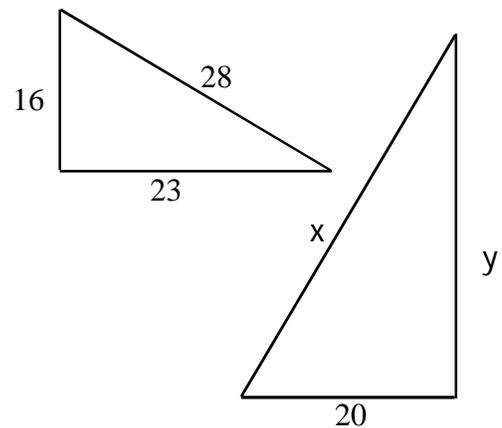
You can use proportions to find the length of missing sides.

Ex. Find the length of sides x and y of the similar triangles below:

1.

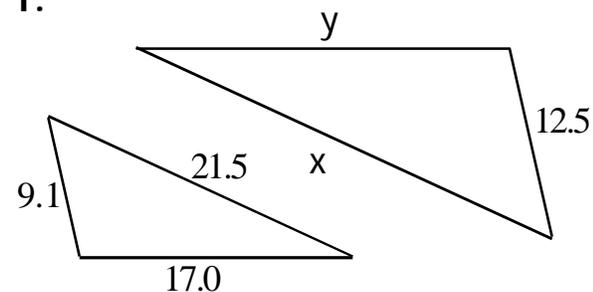


2.

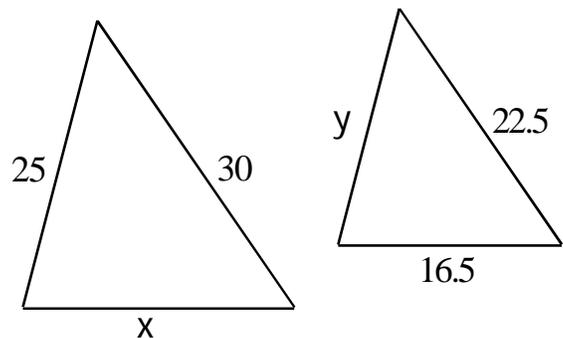


Practice. Find the length of sides x and y of the similar triangles below:
Round to the tenth.

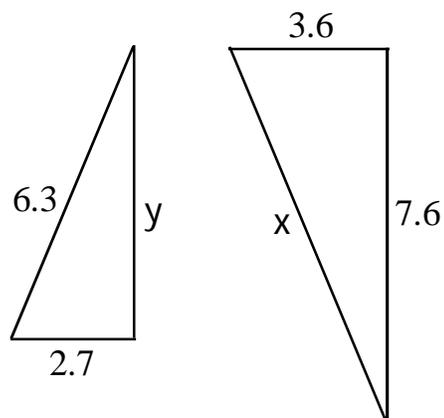
1.



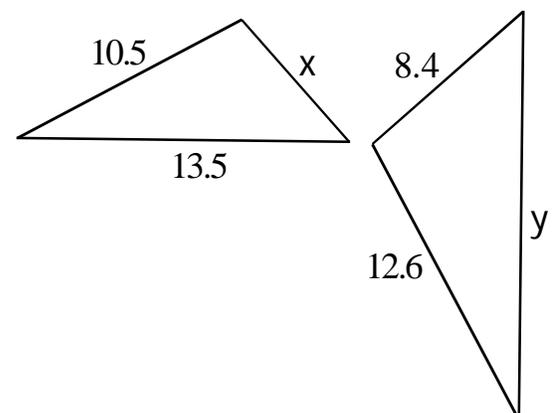
2.



3.



4.



Proportions Review

Algebra

Solve each:

100 What is 20% of 15?

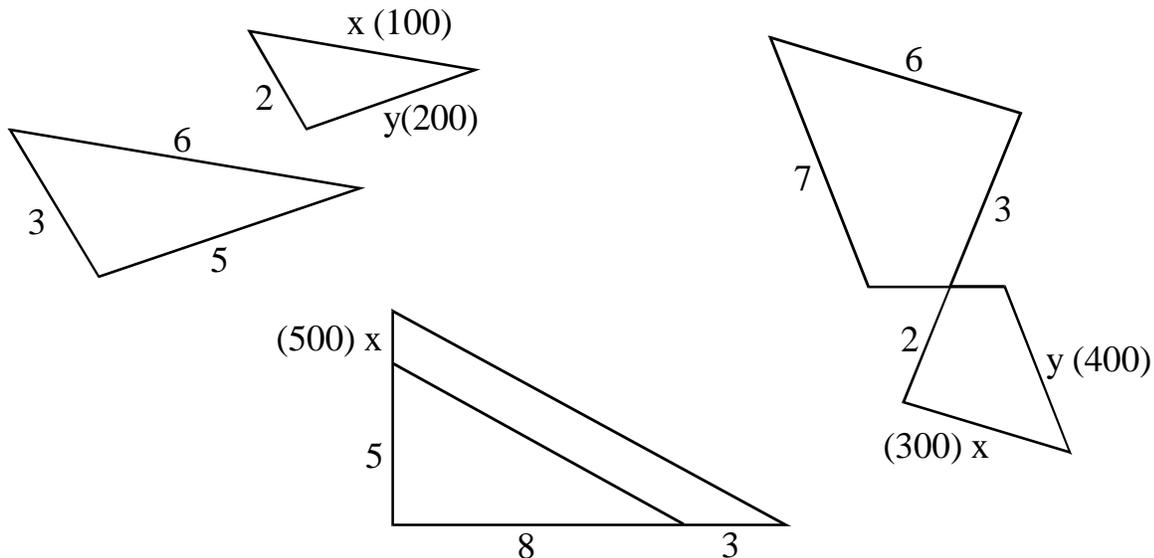
200 You need to get 70 percent of the questions right on a test to pass. How many questions can you miss on a 60-question exam and still pass?

300 What is 30% more than 70?

400 The average gas mileage for small automobiles in America has improved by 15% in the past decade and is now 27.6 mpg. What was the average gas mileage 10 years ago?

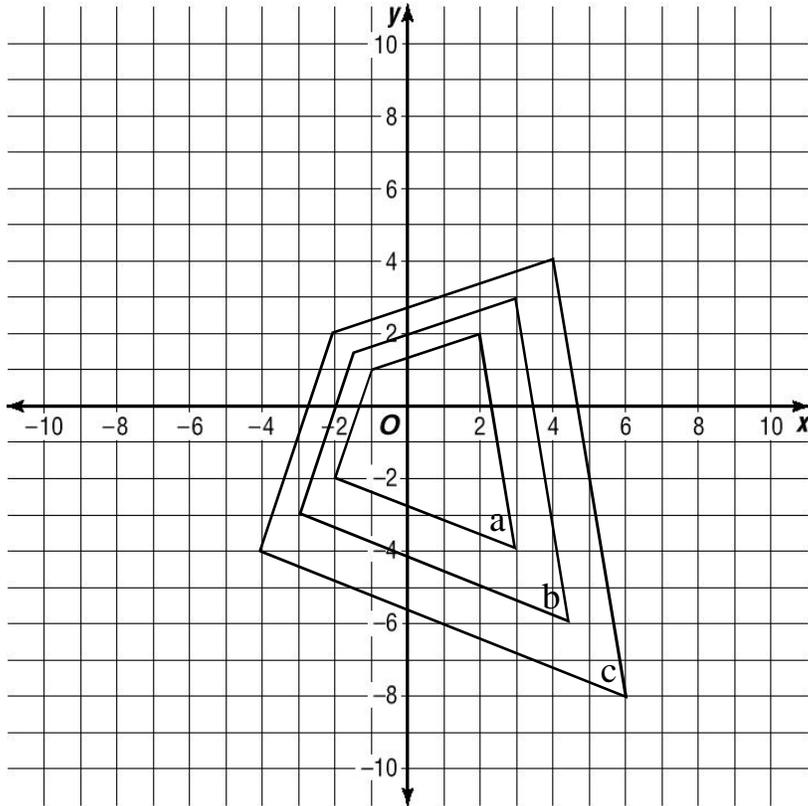
500 Marcia got a 10% raise in 2005, and a 15% raise in 2007. She now makes \$44,275 a year, what was her salary before her two raises?

Find each missing length in the similar figures below:



Proportions Review

Algebra



Find each scale factor:

(100) a to b

(200) a to c

(300) b to c

(400) c to a

(500) c to b

On your graph board with a straight-edge:

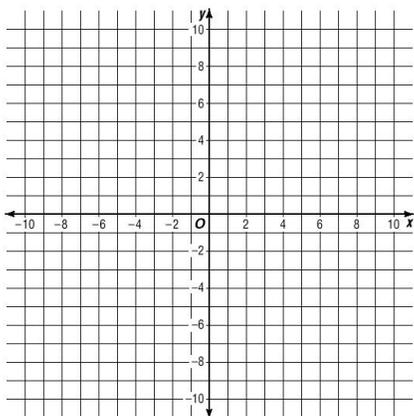
(100) Plot and connect $(0,2)$ $(6,0)$ and $(2,-4)$

(200) Plot a dilation with a scale factor of $1/2$

(300) Plot a dilation of the original with a scale factor of 2

(400) Plot a dilation of the original with a scale factor of $3/2$.

(500) Plot a dilation of the original with a scale factor of $5/3$.

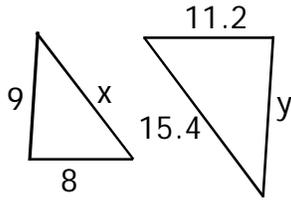


Practice Quiz: Proportions

Algebra 4.4+

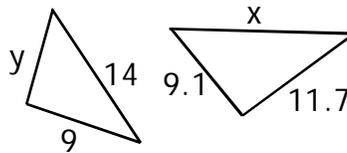
Find the length of the missing sides x and y . Round to the tenth.

1.



1. x _____ y _____

2.



2. x _____ y _____

Solve: Round decimal answers to the tenth.
Use bar notation for repeating decimals. Show all units.

3. What is 15% of 150?

3. _____

4. 10 is what percent of 290?

4. _____

5. 20 is 125% of what number?

5. _____

6. What number is 40% of 12?

6. _____

7. What number is 15% greater than 80?

7. _____

8. 60% less than a number is 33.6. What is the number?

8. _____

9. What number decreased by 70% equals 15?

9. _____

Practice Quiz: Proportions

Algebra 4.4+

Solve: Round decimal answers to the tenth unless noted otherwise. Show all units.

10. A coat is on sale for 30% off. If the original price was \$56.95, what is the sale price to the nearest cent? 10. _____
11. The tax on a \$10.95 pizza is 9%. How much change will you get back if you pay with a \$20 bill? 11. _____
12. The number of students at Ligon decreased by 6% this year. There are 1,021 students at Ligon this year, how many were here last year (to the nearest student). 12. _____
13. A \$65.00 stereo costs \$69.55 after tax is added. What percent is the tax amount? 13. _____
14. A calculator is on sale for 25% off and now costs \$61.74. What was the original price of the calculator to the nearest cent? 14. _____

Find the percent of increase or decrease. Round to the tenth of a percent.

15. New price: \$24.90
Original Price: \$22.91 15. _____
16. New price: \$125.00
Original Price: \$320.00 16. _____

Determine the scale factor used in each dilation below:

17-20.

17. a to b _____

18. a to c _____

19. d to e _____

20. e to d _____