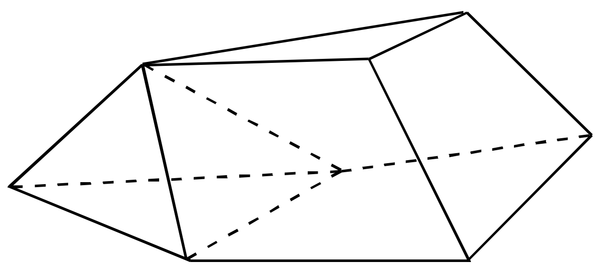
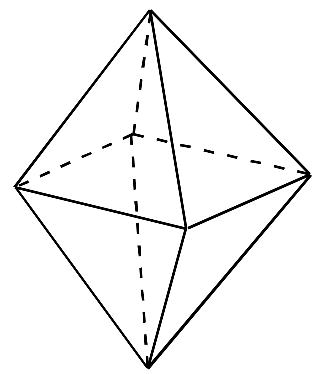
Solid Geometry

Vocabulary and More Drawing!

A **polyhedron** is a geometric solid made up of polygon **faces** which meet at straight-line **edges** that come together at **vertices.**

Like polygons, polyheda are named with prefixes we have already used.
Octahedron = 8 sides.
Hexahedron = 6 sides.
The only exception is the **Tetrahedron**, which has four sides (it is not called a quadrahedron).





Polyhedra can be **regular or irregular**.
Name the two figures above.

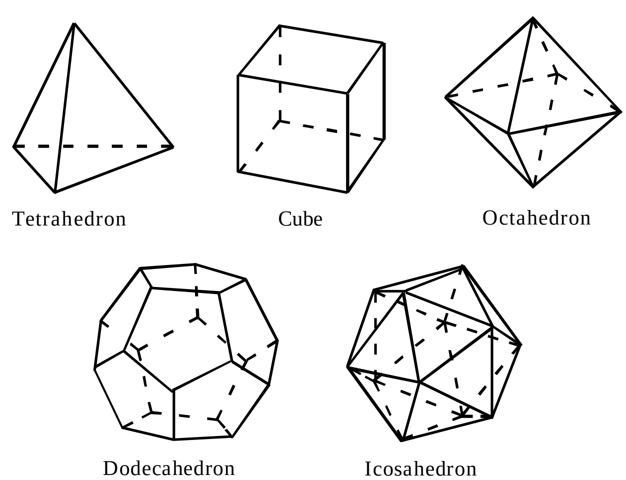
Draw:

1. A regular hexahedron and two irregular hexahedra.

2. A hexahedral pyramid. Give a better name to this figure.

3. A rectangular prism.

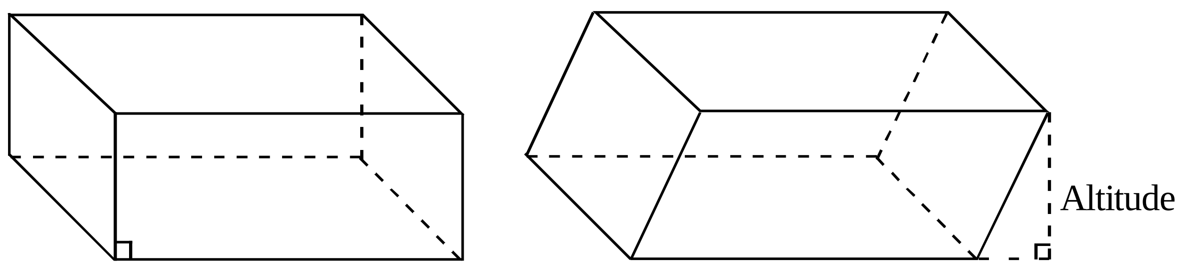
There are five regular convex polyhedra, referred to as the Platonic Solids.



Prisms and Pyramids

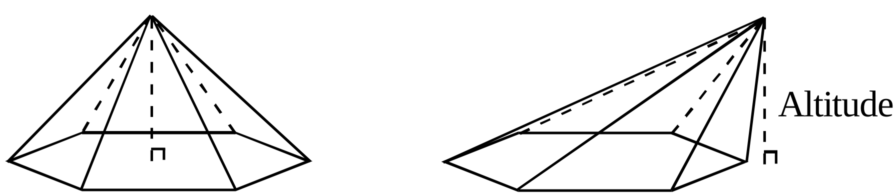
Prisms: Congruent **bases** connected by **lateral faces** which are parallelograms.

If the **lateral faces** are rectangles, the prism is called a **right prism**.
If the **lateral faces** are not rectangles, the prism is called **oblique**.



Pyramids: A single base connected by triangular **lateral faces** which meet at a point.

A pyramid is **right** if the apex is aligned above the centroid of its base.
A pyramid is **oblique** if its apex is not centered above its base.



A **cylinder** is similar to a prism, with a circular base. The line connecting the centers of the bases is called the **axis.** Cylinders may be **right** or **oblique**. In a right cylinder the _______ is perpendicular to the ______.

A **cone** is similar to a pyramid with a circular base. Cones may be **right** or **oblique**. The altitude of a right cone connects the center of the base to its apex. Cones may be **right** or **oblique**.

A **sphere** is the set of all points in space that are a given distance from a given point.

Prisms and Pyramids

Using your notes:

1. Draw and label all parts of an oblique pentagonal prism.

2. Draw and label all parts of a right triangular pyramid.

3. Draw an octahedron which is a pyramid.

4. The faces of a regular dodecahedron are pentagons.
Can you draw one? At least try!

Label each statement as true or false. Provide a sketch to support your conclusion.

1. A decahedron can have two square 'bases' connected by eight triangular lateral faces, similar to a square prism.

2. A regular tetrahedron can be created using triangles or squares.

3. A hexagonal pyramid can be created in which the lateral faces are equilateral.

4. An oblique prism can have six faces which are all parallelograms.

How many:
Edges, faces and vertices?

1. A Hexagonal pyramid.

2. A trapezoidal prism.

3. A cylinder? (What about a cone?? A sphere ???)

4. Eulers formula relates the number of vertices (v), edges (e) and faces (f) of any convex poyhedron. Can you discover Euler's formula?

v-e+f=2

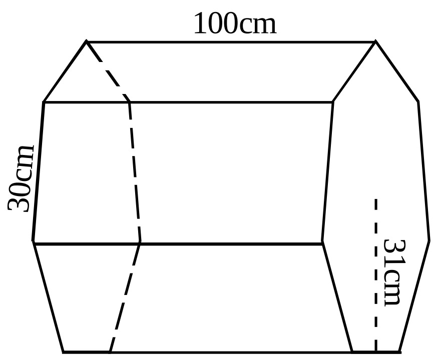
Prism Volume

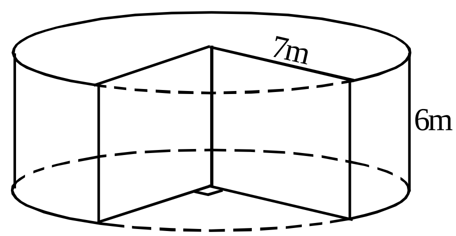
The formula used to find the volume of a prism or cylinder:

$$V = Bh$$

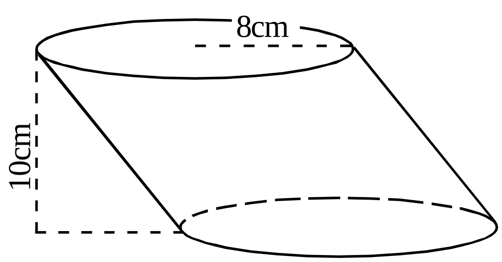
Where B is the area of the base and h is the height. This applies whether the prism is right or oblique. Height is the perpendicular distance between the bases.

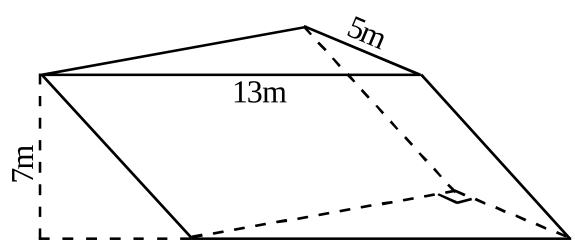
Practice:
Find the volume of each solid.





Practice:
Find the volume of each solid.





Word problems:

1. You are storing leftovers in tupperware containers. You have a 9-inch square pan that is 3 inches deep full of mashed potatoes that you want to store in the refrigerator.

a. How many 2-inch deep cylindrical containers will you need if each has a radius of 3 inches?

b. You also have four-inch square containers that are 2 inches deep. Can you fit all of the potatoes into four square and two round containers?

Pyramid Volume

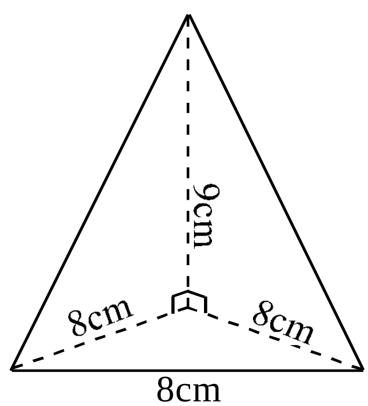
Geometry 10.4

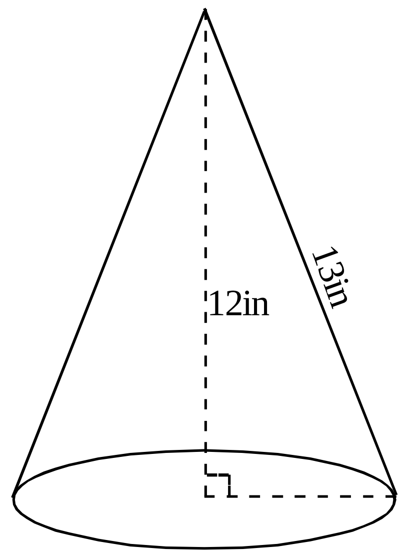
The formula used to find the volume of a pyramid or cone:

$$V = \frac{1}{3}Bh$$

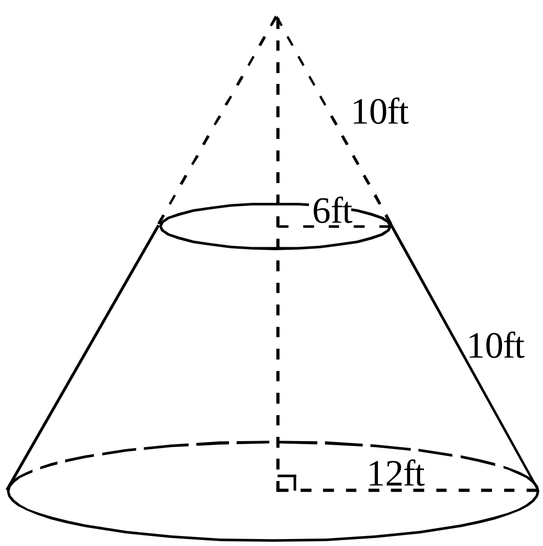
Where B is the area of the base and h is the height.
This applies whether the figure is **right or oblique** (height is measured along the altitude).

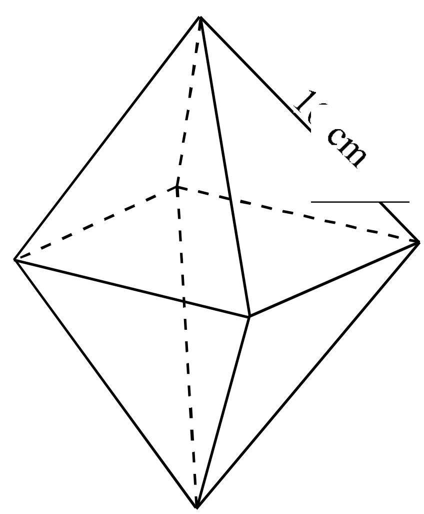
Practice:
Find the volume of each solid.





Practice:
Find the volume of each solid. The octahedron is regular.





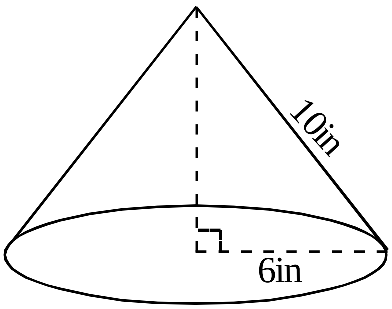
Pyramid/Cone Volume

Geometry 10.4

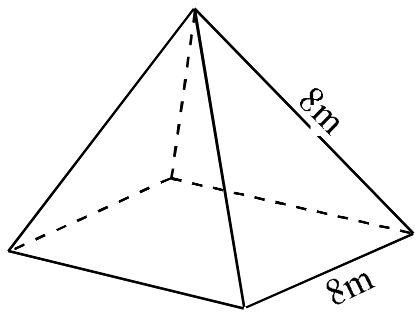
Practice:

Find the height of each, then find the volume.

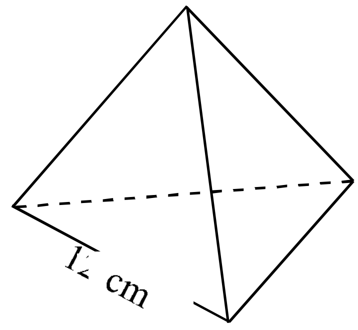
Cone:



Square Pyramid:



Regular tetrahedron:

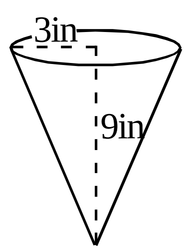


Word Problems:

1. A 5-foot tall cylinder with a 2-foot radius is filled with three feet of water. A steel cube with 18 inch edges is dropped into the cylinder. How many inches does the water level rise?

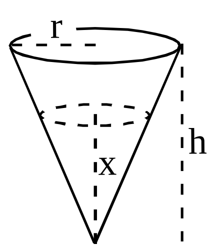
2. Ice cream is sold in a square box that is 3 x 6 x 8 inches. How many cones can be filled (just to the rim) if each has a 1.5-inch radius and is 4 inches tall?

3. The cone below is filled with 6 inches of water. Is this more, less, or equal to half the volume?



Challenge:

How many inches high would you need to fill a cone of height h to fill half the volume?



Faces, Edges, and Vertices

The formula relating the number of faces, edges, and vertices is easy :

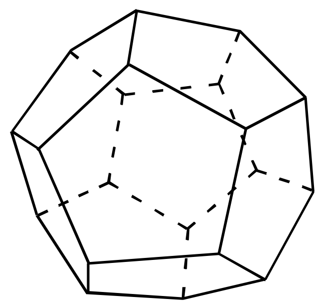
The sum of the faces and vertices is always two more than the number of edges.

$$f+v = e+2$$

You should also recognize that each edge is shared by two faces.

Example:

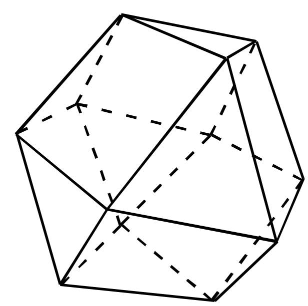
A dodecahedron is made up of twelve regular pentagons. How many edges and vertices does it have?



Practice:

1. A cuboctahedron (A) is made up of eight triangular faces and six square faces. How many edges and vertices does it have?

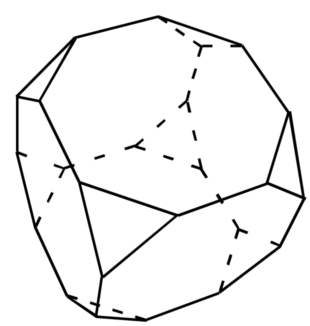
(A)



2. Most soccer balls are constructed of 12 pentagons and 20 hexagons. How many seams must be stiched to sew together the soccer ball.

3. A truncated cube (B) is created by slicing off the eight corners of a regular cube. How many faces, edges, and vertices does it have?

(B)



Name_________________________ Period _____

Practice: F/V/E and Volume Practice

Geometry

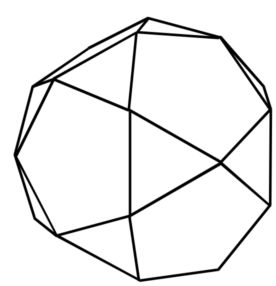
Faces / Vertices / Edges:

1. A snub dodecahedran has 92 faces and a total of 210 vertices and edges. How many vertices does a snub dodecahedron have?

2. A "truncated octahedron" is formed by joining 8 hexagons and 6 squares. What is the sum of the number of vertices, faces, and edges on a runcated octahedron?

3. A "rhombicuboctahedron" is formed by joining squares and equilateral triangles. It has 26 faces and 48 edges. How many of its faces are triangles?

4. An "icosidodecahedron" is made by joining pentagons and equilateral triangles. Each of its 30 vertices is surrounded by two pentagons and two triangles as shown. How many edges does an icosidodecahedron have?



Name_________________________ Period _____

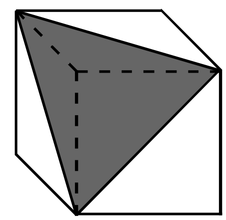
Practice: F/V/E and Volume Practice

Geometry

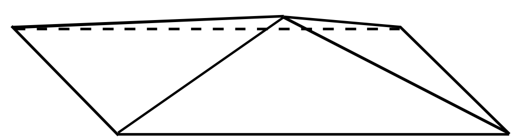
Volume: Levae answers in radical form.

5. A cake is shaped like a cylinder with a height of 4 inches and a diameter of 12 inches. The cake is frosted on the top and around its circumference with a 1/2-inch layer of icing. What is the total volume of the frosting on the cake?

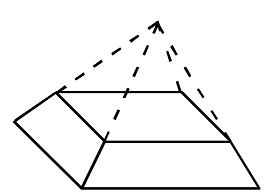
6. One corner of a cube of wood is cut off as shown so that the new face is an equilateral triangle with sides that are the diagonal length of the cube's faces. What is the remaining volume of the cube if the sides of the equilateral triangle are 6cm long?



7. A rectangular pyramid has a base that is 6cm by 10cm and two lateral faces that are equilateral triangles of side length 6cm. What is its volume?



8. A right square pyramid has 8 edges, each 6cm in length. The top of the pyramid is chopped off to create a truncated pyramid whose top is a square of side length 4cm. What is the volume of the remaining truncated pyramid?



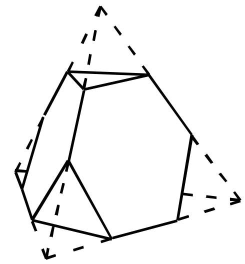
Name________________________ Period _____

Practice Quiz: Solid Geometry

Geometry 10.4

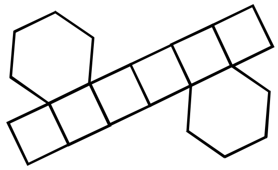
Solve:

1. A truncated tetrahedron is a regular tetrahedron with all four corners cut off. The resulting polyhedron has four triangular faces and four hexagonal faces. How many edges does a truncated tetrahedron have?



1. ___________

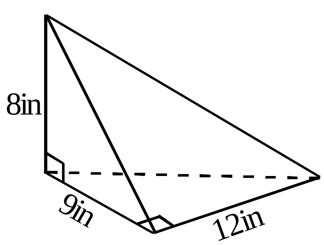
2. The net below is folded to form a polyhedron. How many vertices will it have?



2 . ___________

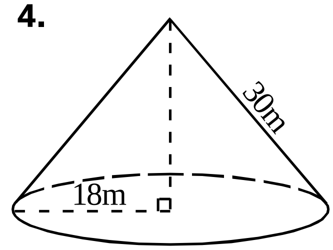
Find the volume of each figure below: Round answers to the tenth. Figures are as they appear (but not to scale), ask if you have questions.

3.



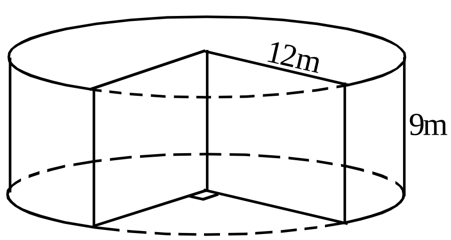
3. ________

4.



4. ________

5.



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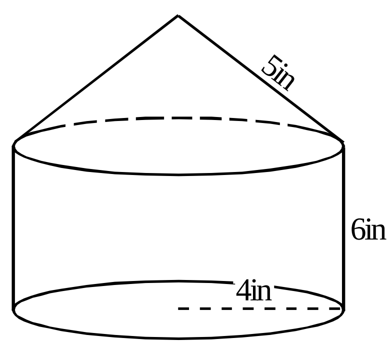


5. ________

6. ________

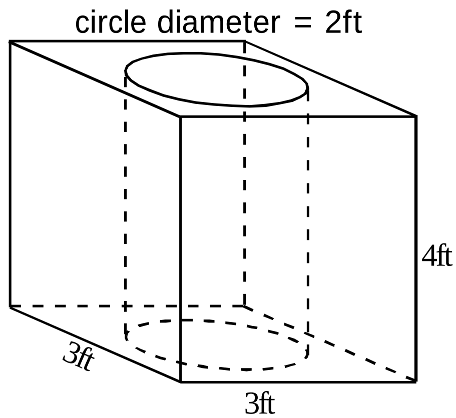
Name_________________________ Period _____

Practice Quiz: Solid Geometry

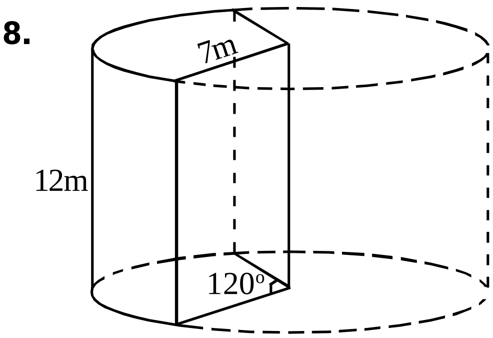
Geometry 10.4

Find the volume of each figure below: Round answers to the tenth.
Figures are as they appear (but not to scale), ask if you have questions.

7.



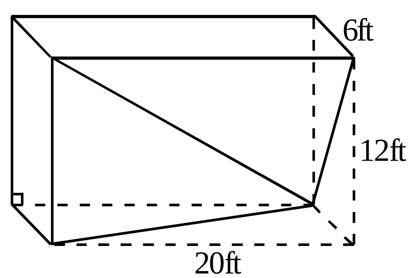
8.



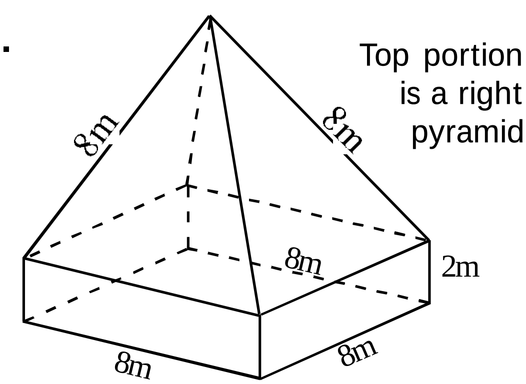
7. ________

8. ________

9.



10.



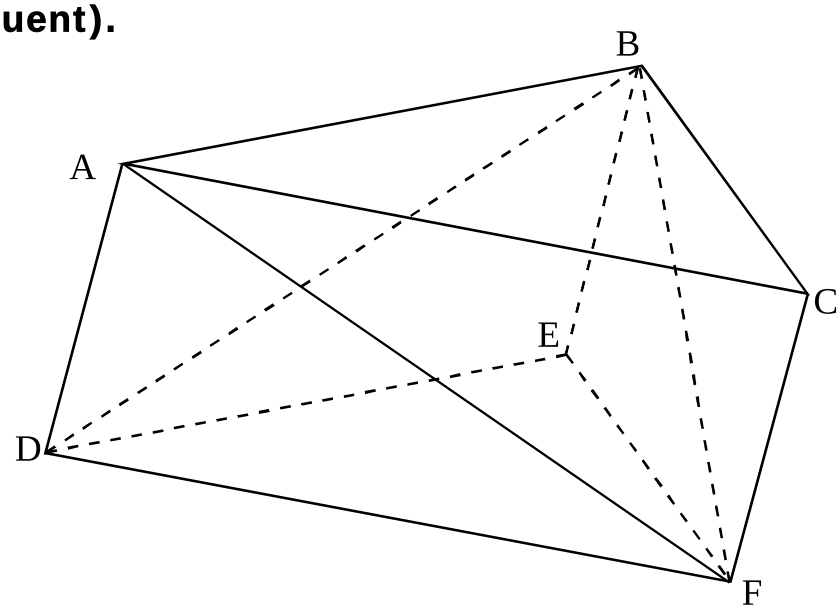
9. ________

10. ________

Euclid's Proof of the Volume Formula for Pyramids:

Geometry 10.4

Given any triangular prism with bases ABC and DEF (even if oblique, you can divide it into three pyramids of equal volume (not congruent).



The pyramid with base ABC is congruent to the pyramid with base DEF (congruent bases and altitudes).

Less obvious is that the pyramid with base ADF and apex B is equal in volume to the pyramid with base ACF and vertex B (which is the same as pyramid with base ABC and apex F). This works even if the prism is oblique, since ACFD is a parallelogram, triangles ADF and FCA are congruent.

Transitivity proves that all three pyramids are congruent.

Since any polygon can be divided into triangles, the same formula works for all pyramids. If you want the proof for cones, you are on your own.

$$V = \frac{1}{3}Bh$$

Displacement and Density

Determining the density of a material can be done using a graduated cylinder and a balance. If an object is more dense than the liquid it is submerged in, it will sink and displace its own volume.

Ex: A 27 gram pebble is submerged in a cylinder with a radius of 1cm, causing the level to rise by 3.1cm. What is the density of the rock in g/cm^3?

Practice:

1. A roll of quarters is 1.5cm in diameter and 8cm tall. You drop a roll of quarters into a cylinder with a 2.5cm diameter. How many centimeters does the water level rise?

The roll of quarters weighs 127 grams. What is its density?

2. A solid gold cube with 2cm edges is dropped into a cylinder of water. The water level in the cylinder rises by 1cm. What is the diameter of the cylinder? (round to the hundredth)

What is the density of gold if the cube weighs 154.4 grams?

3. A graduated cylinder has a diameter of 3cm. How many cm high on the cylinder is the 100mL mark? (note: 1mL = 1cc = 1cm^3, round to the hundredth.)

If an object floats, determining its density is more difficult.
Here is a counter-intuitive question:
A large block of ice floats in a cooler full of water so that some of the ice is above the water and some is below. You place a mark on the cooler to note the water level. After a day, the ice has melted. Is the water level higher or lower than before? (assume no evaporation has occured)

1. Water has a density of 1g/cm^3. If a rubber ball floats half-submerged, what is its density?

2. A meterstick is floating upright so that 76cm are submerged (leaving 24cm above water). What is the density of the meterstick?

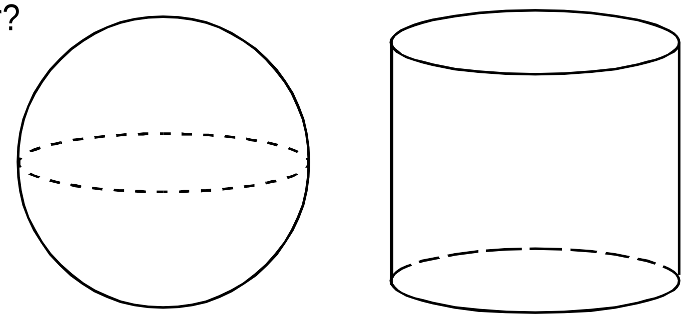
3. The meterstick above is 2cm wide and 1/4 cm thick. How much does it weigh (in grams)?

Volume of Spheres

The formula for volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

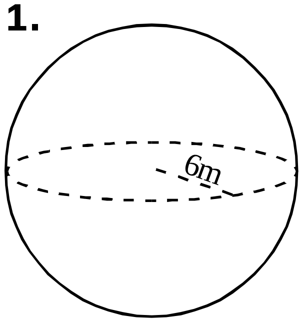
Assume the sphere and cylinder below have the same radius and height: What is the relationship between the volume of the sphere and the volume of the cylinder?



Practice:

Determine the volume of each:

1.



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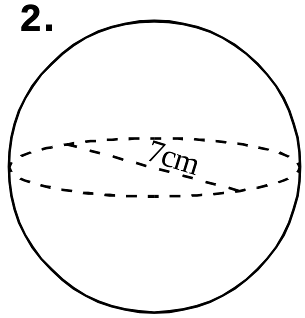

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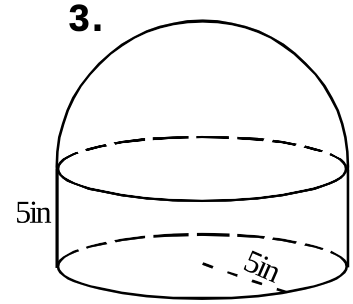


Practice:

1. A sphere is carved from a cube of wood whose edges measure 18 inches. How many cubic inches of wood must be carved away?

2. An ice cream cone is 4 inches tall and has a 2.5-inch radius, packed and topped with a hemisphere of ice cream. You also have the option of buying a 3-inch tall cup of ice cream with a 2-inch radius. Which option gives you more ice cream?

3. When a styrofoam sphere with a 30-cm diameter is dropped into a cylinder of water with a 20-cm radius, the water level rises by only one centimeter. What is the density of the sphere to the hundredth of a g/cm^3? (Similar question: What percent of the sphere is underwater?)

Name________________________ Period _____

Volume of Spheres

Geometry 10.6

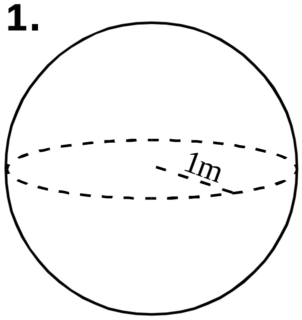
The formula for volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

Practice:

Determine the volume of each:

1.



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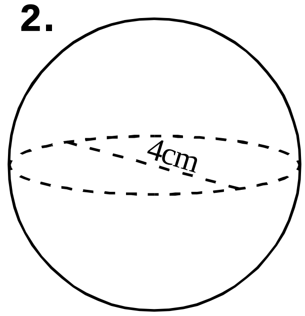

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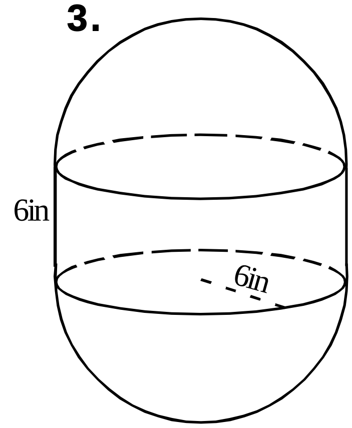
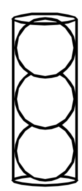


Practice: Solve each. Answers should be left in terms of pi.

4. The volume of a sphere is $\frac{16\pi}{3}cm^3$. What is its radius?

4. __________

5a. Three tennis balls are sold in a cylinder. The balls have a 3-inch diameter. If the cylindrical container is made just large enough to hold three balls, what will be the volume of the *remaining space* in the cylinder?



5a. __________

5b. What fraction of the cylinder is filled by the tennis balls?

5b. __________

6. A cube has 2-inch edges and is inscribed within a sphere. Find the volume of the sphere (in radical form).

Think... how could you find the diameter/radius of the sphere?

6. __________

7. A cylinder is inscribed within a sphere. The cylinder is 6 inches wide and 8 inches tall. What is the volume of the sphere?

Think... how could you find the diameter/radius of the sphere?

7. __________

Name________________________ Period _____

Volume Review

Geometry 10.6

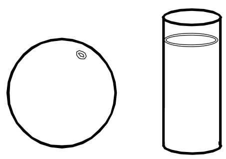
Practice: Solve each. Answers should be left in terms of pi or in radical form.

8. A cylinder and a sphere each have a 9” radius and have equal volumes. How many inches tall is the cylinder?

8. __________

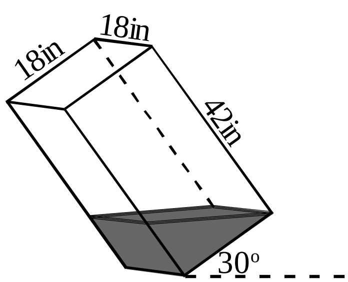
9. A mathland grapefruit has a 6” diameter. Assume that the grapefruit is 50% juice. If you squeeze all of the juice out of the grapefruit, how many inches high will the juice fill a cylinder glass with a 3” diameter?

9. __________



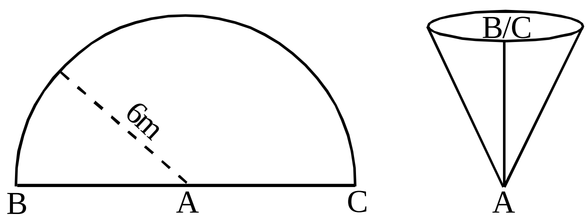
10. How many cubic inches of water are in the tilted rectangular prism below?

10. __________



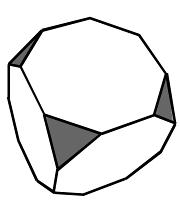
11. The semicircle below is rolled into a cone with apex A by rolling and connecting AB to AC. What is its volume?

6. __________



12. A wooden cube has 6cm edges. Each of the eight corners is sliced off so that the resulting faces are equilateral triangles whose edges are 2cm long. Express the volume of the resulting truncated cube in radical form.

7. __________



Name________________________ Period _____

Displacement & Density

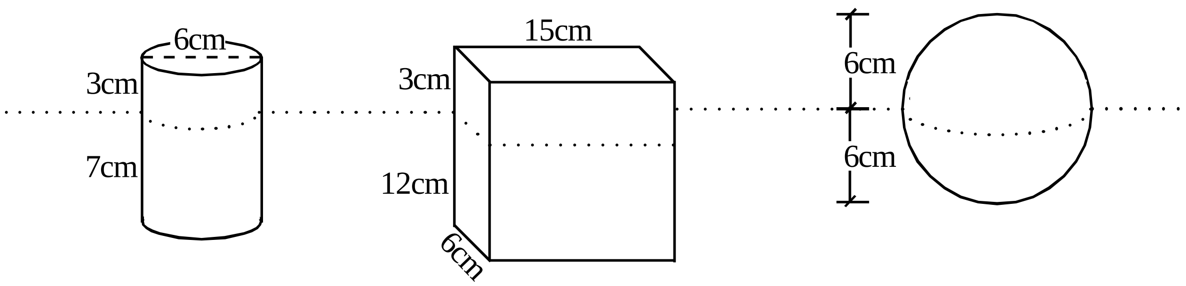
Geometry 10.6

Density: $d = \frac{m}{v}$ **Some important background information:**

The density of water at 40ºF is 1.0g/cm^3. Use this for your calculations below.
Objects with density less than 1g/cm^3 float.

Example: If the density of an object is .75g/cm3,
then 75% of the object will be under water.
If the density of an object is .35g/cm3,
then 35% of the object will be under water.

Practice: Determine the density of each object below: The dotted line represents the water line, the measurements show the distance above and below the water line.



1.____g/cm^3 2.____g/cm^3 3.____g/cm^3

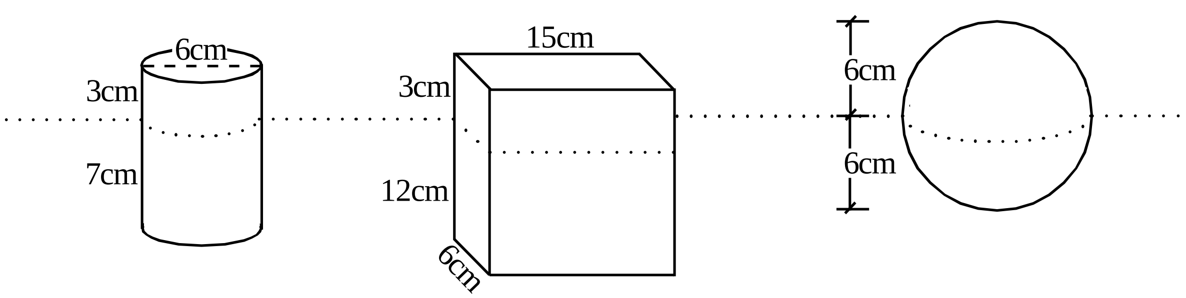
Any object with a density greater than 1g/cm^3 will sink in water.

Mass: $m = dv$ Now, determine the mass of each object above.

4.____grams 5.____grams 6.____grams

Displacement: An object displaces exactly the same volume of water as the volume of the submerged portion of the object.

Easy practice: Determine volume of water displaced by each of the objects below.



7.____cm^3 8.____cm^3 9.____cm^3

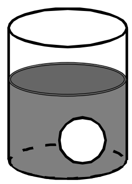
Name________________________ Period _____

Displacement & Density

Geometry 10.6

Harder Practice: Leave answers in pi/radical form or round as noted.

10. A cannonball with a 4-inch radius sinks to the bottom of a barrel full of water, raising the water level by 1/3 of an inch. What is the radius of the cylindrical barrel?

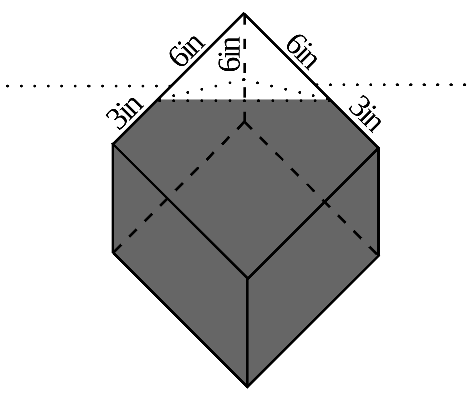


10. __________inches

11. A 6-inch cube of wood floats in a fish tank with 12” x 24” base, raising the level of the water by 3/8”. What is the density of the cube of wood in g/cm^3?

11. __________g/cm^3

12a. A cube of wood floats with one corner out of the water as shown. What fraction of the cube is *under* water?

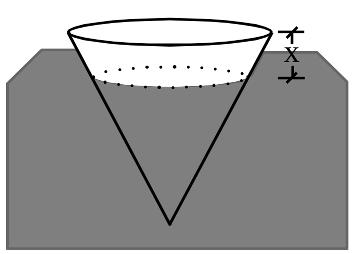


12a. __________

12b. What is the density of the cube (rounded to the hundredth)?

12b. __________g/cm^3

13. A cone has a 6cm radius and is 8cm tall, with a density of .5g/cm^3.
What is the height of the cone above the water (to the hundreth)?



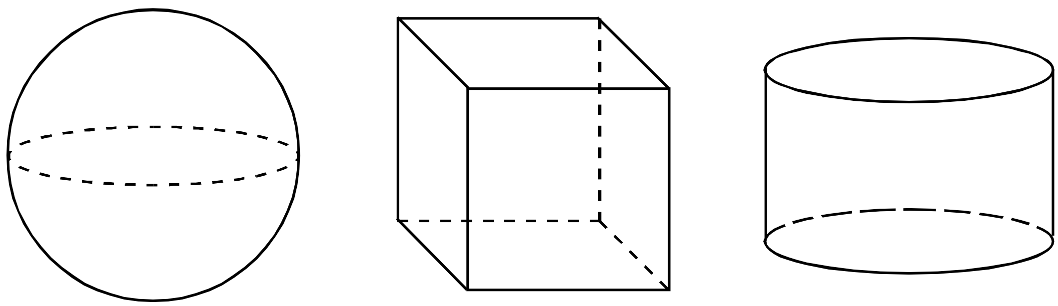
13. __________cm

Surface Area of Spheres

The formula for surface area of a sphere:

Surface Area = $4\pi r^2$

The sphere, cube, and cylinder below have the same volume. The cylinder and sphere share the same radius of 5cm.

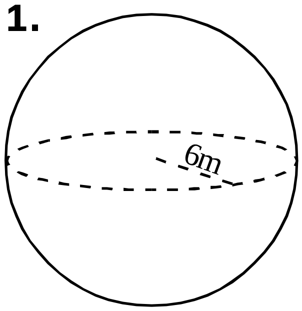


1. How tall is the cylinder?
2. To the hundredth, how long is each square side of the cube?
3. List all three in order from greatest to least in terms of surface area.
4. You want to design a space capsule which retains heat, but still has plenty of room (note: more surface = more heat loss). What shape makes the most sense for this application?

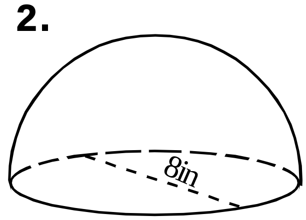
Practice:

Determine the surface area of each:

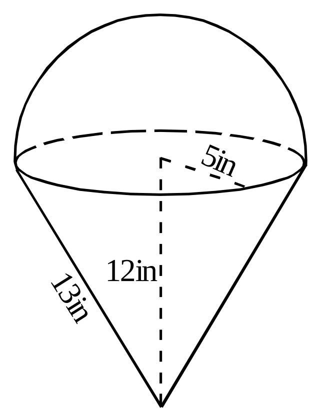
1.



2.



3.



Practice:

Find each surface area:

1. A sphere with a volume of $70m^3$.

2. A hemisphere with a volume of $8cm^3$.

3. A sphere that weighs 20 grams and has a density of $3g/cm^3$.

Name________________________ Period _____

Surface Area Practice

Geometry 10.6

Harder Practice: Leave answers in pi/radical form or round as noted.

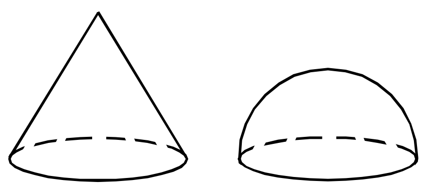
1. What is the formula for the surface area of a hemisphere (including the bottom), given the radius *r* ?

1. _____________

2. The volume of a sphere is equal to its surface area. What is its radius?

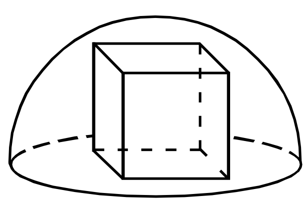
2. _____________

3. A cone and a hemisphere each have a 3-inch radius and the same surface area. What is the height of the cone?



2. _____________

4. A cube is inscribed within a hemisphere. The cube has 2-inch edges. What is the surface area of the hemisphere?



4. __________

5. A cube has a spere inscribed within and a sphere circumscribed about it. What is the ratio of the surface area of the inscribed sphere to the surface area of the circumscribed sphere?

5. ________

Name________________________ Period _____

Volume Review & Practice

Geometry 10.6

Practice: Leave answers in pi/radical form or round as noted.

1. A cube of wood has 3cm edges and has a mass of 30 grams. What is its density? Will it float?

1. ______________ Y / N

2. A sphere has a surface area of 36π cm^2. What is its volume?

2. ______________

3. A fish tank has a base that is 30cm by 50cm and is 40cm tall. You add 20 fish to the tank, each weighing about 25 grams. How much will the water level rise in the tank?

3. ______________

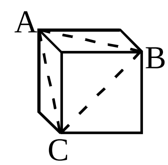
4. A cylinder holds a cubic foot of water. The diameter and height of the cylinder are equal. To the tenth of an inch, find the radius of the cylinder.

4. ______________

5. Two cylinders are partially filled with water. The smaller cylinder has a radius of 3cm and the larger one has a radius of 4cm. If each cylinder is filled to a depth of 24cm of water, how many cm (of depth, not cubic cm) must be poured *from the larger cylinder* into the smaller one so that each has an equal amount of water?

5. ______________

6. The perimeter of equilateral triangle ABC inscribed in the cube below is 6in. What would be the surface area of a sphere circumscribed about the cube?



6. ______________

7. A regular tetrahedron (4 equilateral triangle faces) has an edge length of 6cm. What is the surface area of the largest cone that can be inscribed within the regular tetrahedron?

7. ______________

Name________________________ Period _____

3D Geometry Review & Practice

Geometry

Practice: Leave answers in pi/radical form or round as noted.

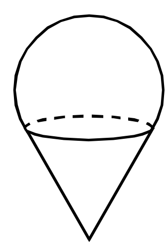
8. A polyhedron has 14 faces and 36 edges. Some of the faces are hexagons and others are squares. How many of the faces are hexagons?

8. ____________

9. A golf ball sinks to the bottom of a cylindrical glass of water of radius 4.26cm. The radius of the golf ball is exactly half the radius of the glass. How many cm does the water in the glass rise? Express your answer as a decimal rounded to the hundredth.

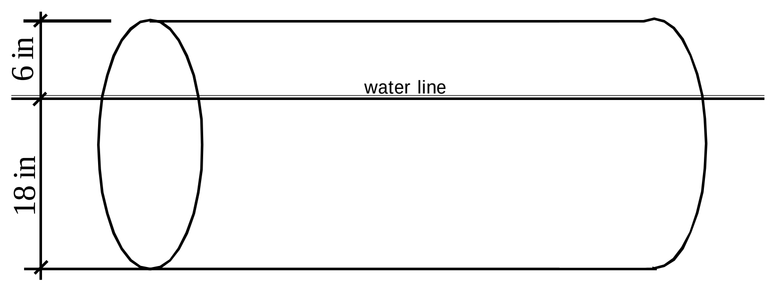
9. ____________

10* A snow cone has a paper cup in the shape of a cone with a slant height of 6cm and a radius of 3cm. The sides of the paper cup are tangent to the sphere of snow as shown. What is the volume of the spherical snowball in the snow-cone cup?



10. ____________

11* A log floats as shown in a river. What is its density in g/cm^3?



11. ____________

hint: 0.75 is incorrect

* hint: you will use a 30-60-90 triangle to solve this problem correctly.

Name________________________ Period _____

Volume Review

Geometry 10.6

Solve each: Round decimal answers to the hundredth.

1. A rubber ball weighing 50 grams is placed into a tank that has a 40x10 cm rectangular base, raising the water level by 0.1cm.

a. What is the density of the ball?

1a. ______________g/cm^3

b. What is the radius of the ball? _____

1b. ______________cm

2. The surface area of a hemisphere is 235.6 cm^2 (including the bottom surface). What is its volume?

2. ______________cm^3

3. Two dozen identical marbles, are needed to raise the water level in a glass of water one centimeter. Each marble has a diameter of 1cm.

a. What is the radius of the glass?

3a. ______________cm

b. If each glass is 15cm tall, how many glasses could be filled with a 2-liter bottle of soda? (1 liter = 1000cm^3).

3b. ______________

4. The circumference of a basketball hoop is 56.6 inches. The surface area of a basketball is 250 in^2.

a. What is the diameter of a basktball hoop?

4a. ______________in

b. What is the diameter of a basktball?

4b. ______________in

c. Based on the above, will two regulation basketballs fit through a regulation hoop side-by-side?

4c. _____(y/n)

d. A fully inflated basketball has a density of .06oz/in^3. How much does a basketball weigh (in ounces)?

4d. _____ounces

Name________________________ Period _____

Volume Review

Geometry

Solve each: Round decimal answers to the hundredth.

5. A gallon is .133 cubic feet. You want to design a spherical container that holds a gallon, and a cylinder with the same radius that holds a gallon.

a. What radius (in inches) should you use?

5a. _____________in

b. What will the height (in inches) of the cylinder be?

5b. _____________in

6. A gallon of water weighs about 8.35 pounds. Approximately 60% of the weight of a healthy human body is water.

a. How many gallons of water are there in a healthy person that weighs 175 pounds?

6a. _____________gal

b. Your science teacher wants to represent the amount of water in a 175-pound human by filling-up a cylindrical trash can that has a radius of 8 inches. To what depth (in inches) should she fill the trash can to accurately represent the amount of water in a 175-pound human? (Use info from #5).

6b. _____________in

7. A freight train has 40 tanker cars like the one below, all transporting grape jelly. A gallon of grape jelly weighs 8 pounds. Each tanker car is a cylinder with a height of 8 feet and a length of 80 feet.

a. How many cubic feet can be transported by one tanker car?

7a. _____________ft^3

b. How many gallons can be transported by a tanker car?

7b. _____________ft^3

c. How many tons does the jelly in the train weigh? (2000lbs = 1 ton)

7c. _____________ft^3

d. A jar of grape jelly has a 3-inch radius and is 5 inches tall, and is enough for 15 pb&j sandwiches. How many pb&j sandwiches can be made with the train full of jelly?

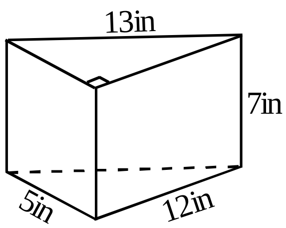
7d. ____________________

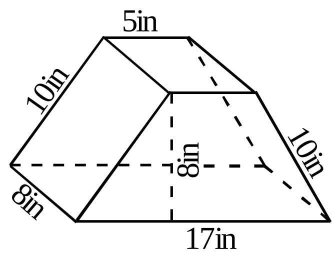
Name________________________ Period _____

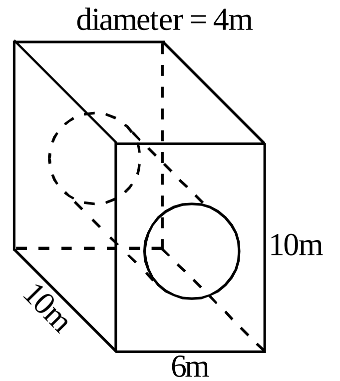
Practice Test Volume

Geometry 10.7

Determine the volume for each: Round to the tenth.



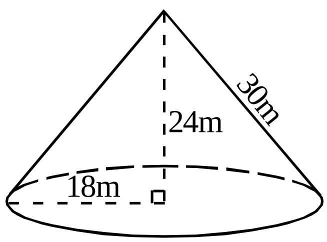


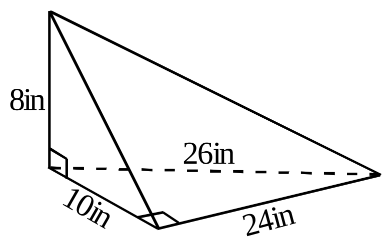


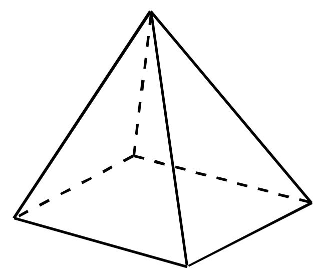
1. V = ________

2. V = _______

3. V = ________







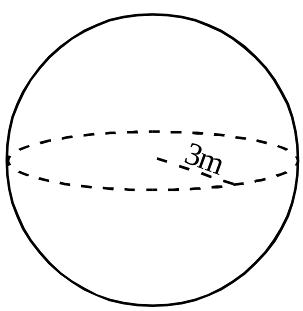
All edges 8cm

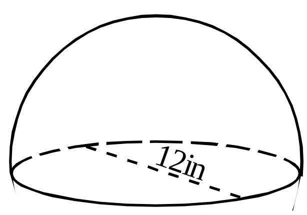
4. V = ________

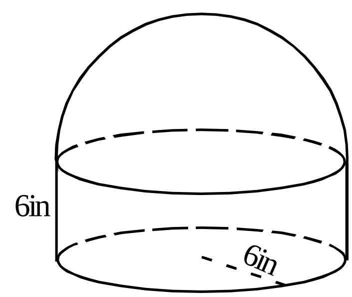
5. V = _______

6. V = ________

Determine the volume and surface area for each: Round to the tenth.







7. V = ________

8. V = _______

9. V = ________

10. SA = ________

11. SA = _______

12. SA = _______

Name_________________________ Period _____

Practice Test Volume

Geometry

Solve each:
Round decimal answers to the hundredth.

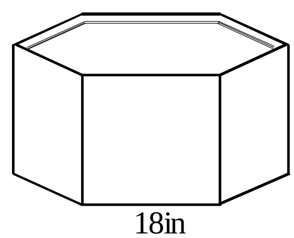
13. A steel cube with 7-inch edges is dropped into a cylinder of water, causing the level to rise 3 inches. What is the the radius of the cylinder?

13. ___________

14. A wooden dowel (cylindrical rod of wood) is 15cm long with a 1.5cm radius. The dowel weighs 77.7 grams. If the dowel floats upright, how many centimeters will be above the surface of the water?

14. ___________

15. A cubic foot of water is about 7.5 gallons. The hexagonal fish tank below holds 65 gallons of water. How many inches deep is the water in the tank?



15. ___________

16. A sphere with a density of $2.4g/cm^3$ weighs 90 grams. What is its surface area?

16. ___________

17. A dodecahedron has 12 faces, each of which is a regular pentagon. How many vertices are there on a dodecahegron?

17. ___________

note: The actual test will have ten or eleven questions.