Similarity

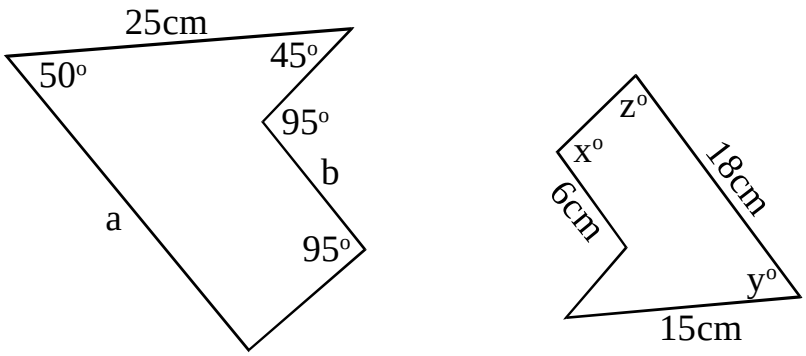
Geometry 11.2

What makes two polygons similar?

Corresponding angles must be _____.
Corresponding sides must be _____.
Basically: Same shape, different size.

Are both definitions necessary?
Try to draw two polygons with congruent angles that are NOT similar.
Try to draw two polygons with proportional sides that are NOT similar.

Similarity can be used to find missing sides and angles of polygons:
Solve for a, b, x, y, and z in the similar pentagons below.



Triangle Similarity
Because triangles are 'rigid', there are similarity shortcuts for triangles (Just as there are congruence shortcuts… name them all.)

In triangles, if all three angles are equal, the sides will always be proportional. Consider a 30-60-90 right triangle. Is it possible to change the length of one side and maintain all three angle measures?

How many angles do you need to determine similarity?

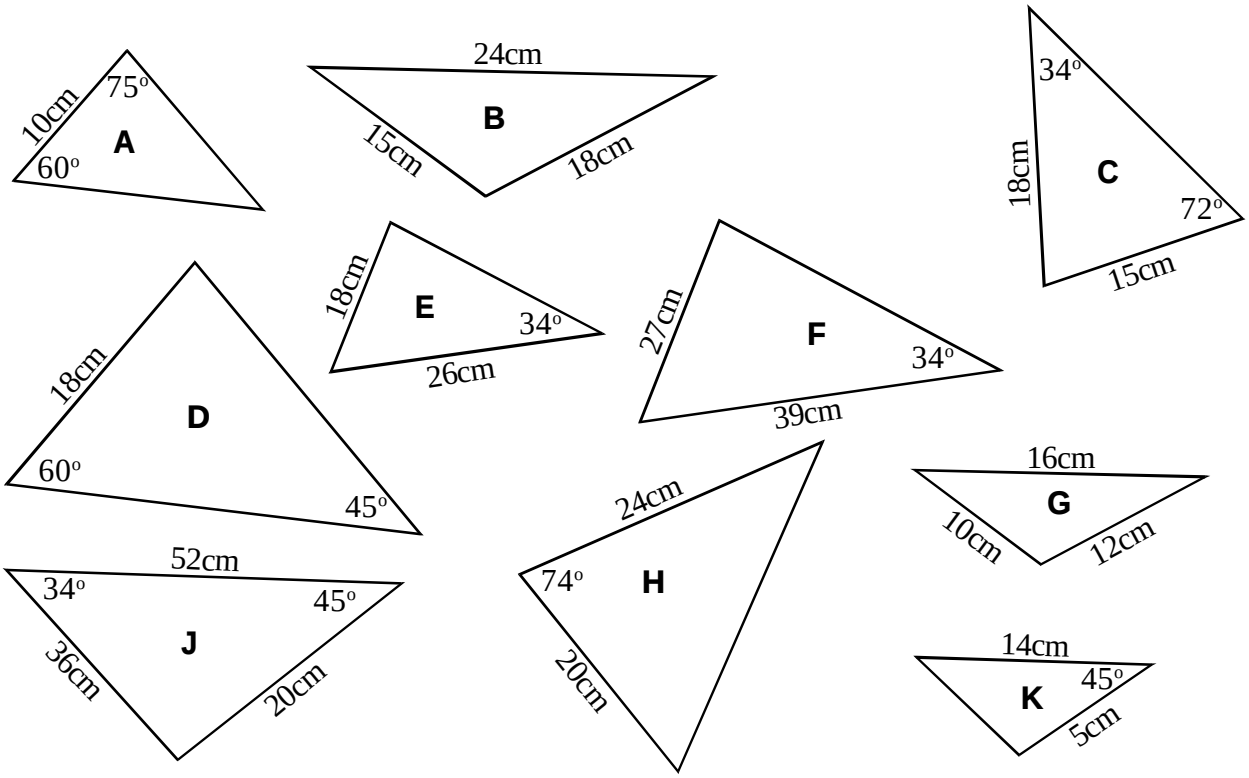
Since AA is a similarity shortcut, so are AAA, ASA, and AAS.
SAS is a similarity shortcut.
SSS is also a shortcut. Consider again two right triangles for demonstration: A 3-4-5 right triangle and a 6-8-10 right triangle. If all three sides are proportional, two triangles are similar.
SSA is not a shortcut for congruence, and cannot be used to prove similarity either because multiple triangles can be drawn with SSA 'similarity'.
AA (AAA) is the only similarity shortcut that is not a congruence shortcut.

Similarity Shortcuts

Geometry 11.2

Practice:

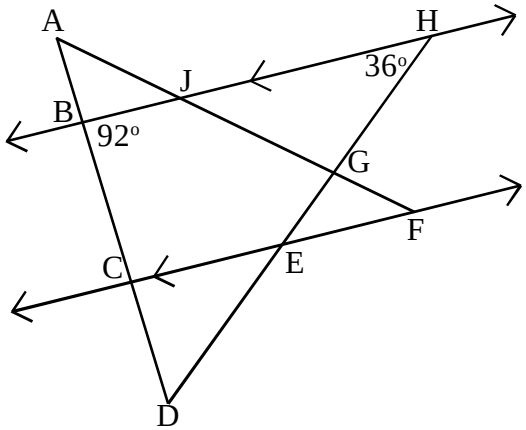
Determine which pairs of triangles below can be proven similar and by which shortcut:

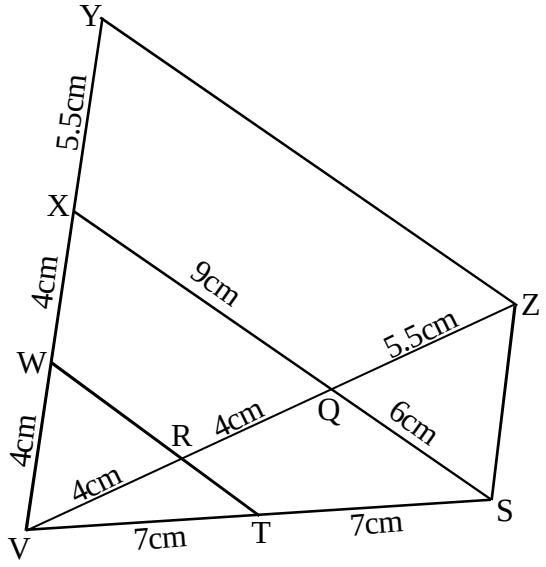


AD by AA, BG by SSS, HC by SAS, E not to F (SSA), J K have no partner

Practice:

Name all the similar triangles in each of the diagrams below:





Indirect Measurement

Geometry 11.3

Shadow proportions:

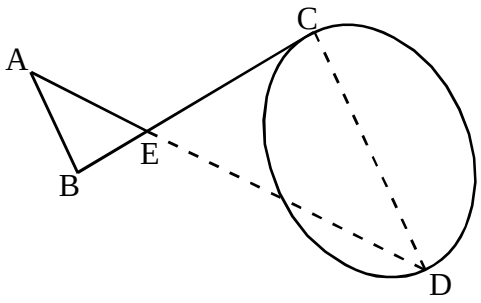
1. The Sears Tower in Chicago is our nations tallest skyscraper. If the Sears tower casts a shadow that is 1,160 feet long while a parking meter that is 5 feet tall casts a 48-inch shadow, how tall is the Sears Tower?

2. Nick's shadow is 5 inches longer than Smirthi's shadow, but he is 7 inches taller than she is. If Smirthi is 59 inches tall, how long is her shadow? (to the tenth)

Sight Lines and Distant Objects

1. You are in a Washington state park at a sign that points out major mountain peaks in the distance. It includes arrows pointing to Mount Rainier 72 miles away, and Mount St. Helens 55 miles away. You just happen to have three metersticks available. Explain how you could use them to determine the distance between the two peaks.

2. You and a friend want to know the exact distance across a large lake. You use stakes, string (dark lines), and sight lines (dashed) to create the following setup:



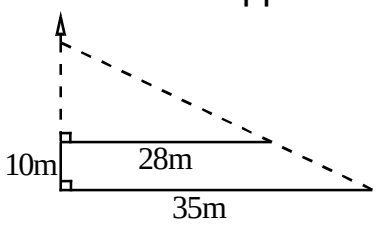
What could you do to make AB and CD parallel?

What measurements would you need to determine the distance across the lake?

Practice:

1. You want to know the height of a flagpole (but it is cloudy). You hold a 6cm ruler vertically 45cm in front of your eye so that the top of the flagpole aligns with the top of the ruler, and the base of the flagpole aligns with the base of the ruler. What additional measurement(s) can be made to determine the height of the pole?

2. Uisng stakes and a tape measurer, explain how you could determine the exact distance across a river to a tall tree on the opposite bank.



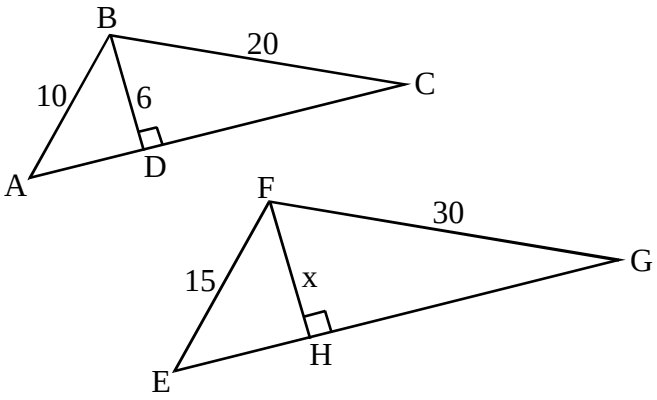
Corresponding Parts

Geometry 11.4

Corresponding Parts of Similar Triangles:
If a pair of triangles is similar (corresponding sides proportional) then the corresponding altitudes, medians, and angle bisectors will also be proportional.

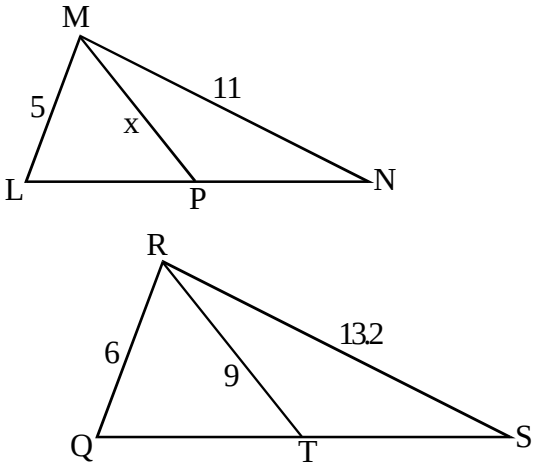
Solve: Solve for the length x in each pair of similar triangles below

1. Altitudes.



How can you show
$\Delta ABD \sim \Delta EFH$?

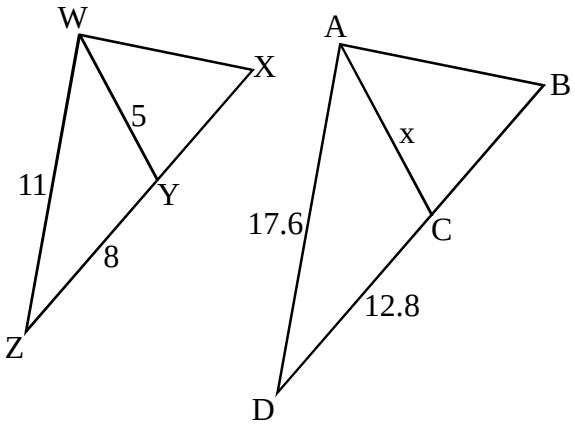
2. Medians



How can you show
$\Delta LMP \sim \Delta QRT$?

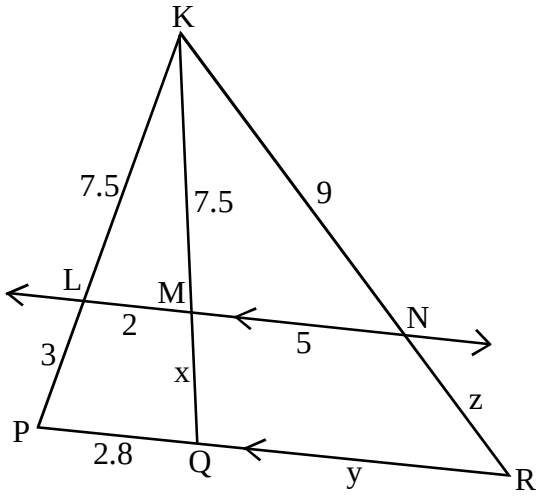
Solve: Solve for the length x in each pair of similar triangles below.

1. Angle bisectors.



How can you show
$\Delta WZY \sim \Delta ADC$?

2. Solve for x, y, and z.

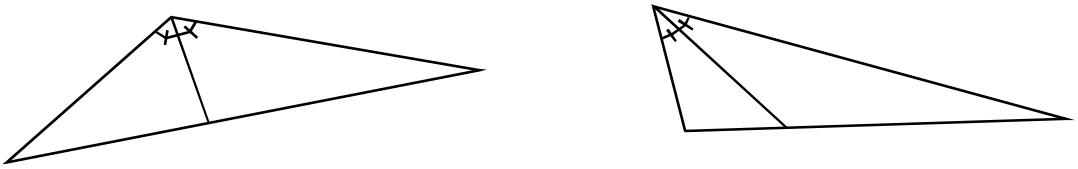


Name all pairs of similar triangles.

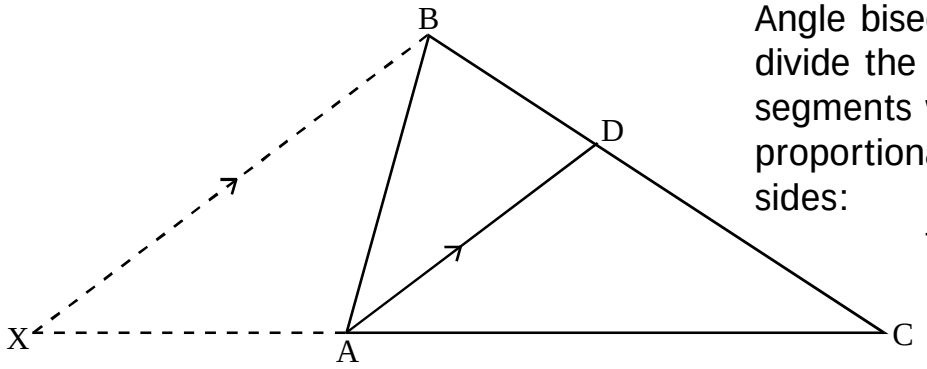
Angle Bisector Theorem

Geometry 11.4

Angle Bisectors



Angle bisectors do not bisect the opposite side of a triangle, but there is a special property of angle bisectors you should understand:



Angle bisectors in a tiangle divide the opposite side into segments whose ratio is proportional to the adjacent sides:

$$\frac{AB}{AC} = \frac{BD}{DC}$$

Proof:

Add point X so that XB||AD.

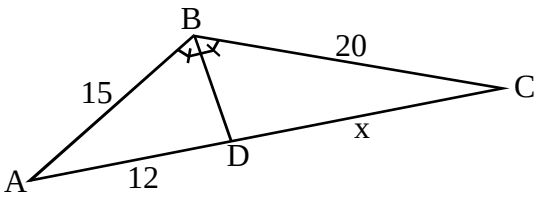
This creates similar triangles DAC and BXC such that $\frac{AX}{AC} = \frac{BD}{DC}$

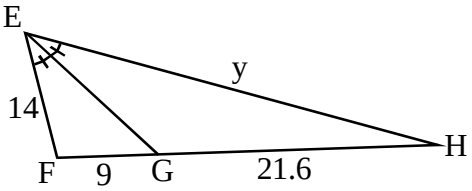
Some simple work with the angles shows that ∠AXB = CAD = BAD = ABX, which means that triangle ABX is isosceles and $\overline{AB} = \overline{AX}$.

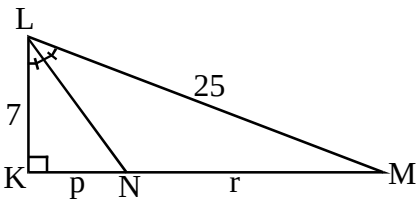
Substitution yields: $\frac{AB}{AC} = \frac{BD}{DC}$

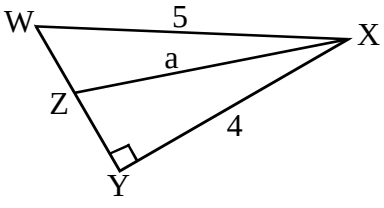
Practice:

Find the missing lengths in the figures below: Round to the hundredth.









Name_____ Period _____

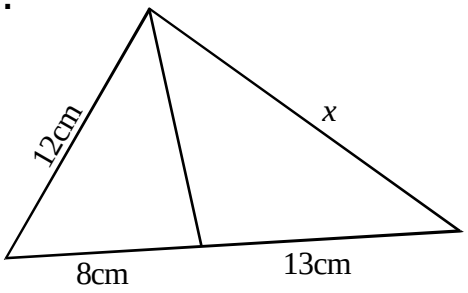
Angle Bisector Theorem

Geometry

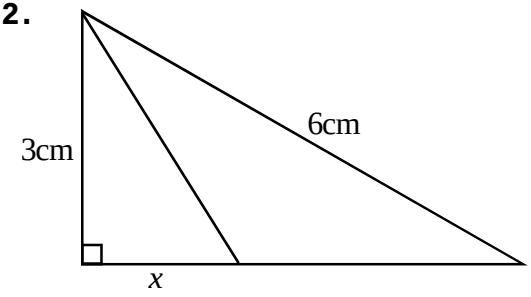
In each diagram, the cevian (line from a vertex to the opposite side) is an angle bisector. Find the missing length x in each diagram.

Round decimal answers to the hundredth or leave them in radical form.

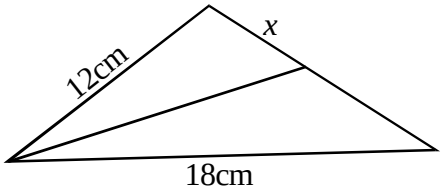
1.



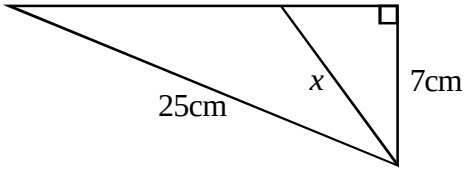
2.



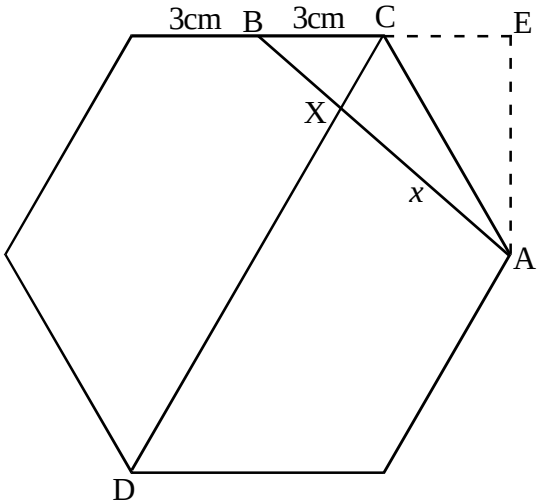
3. (isosceles triangle)



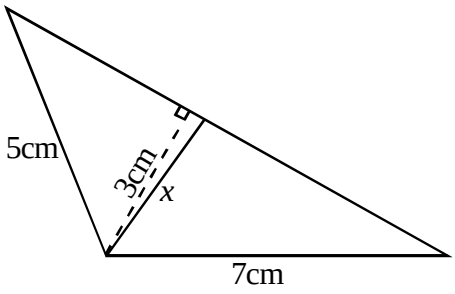
4. (x is the length of the angle bisector)



5. (The hexagon is regular. Find AX.)



6. (x is the length of the angle bisector, round to the thousandth)



Hint: Use similar triangles to find the length of $\overline{AB}$.
$\overline{CX}$ is an angle bisector of $\angle ACB$.

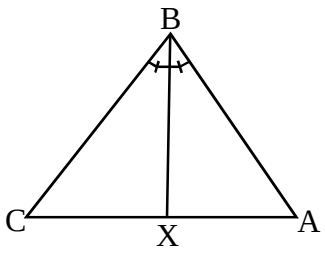
Name_____ Period _____

Angle Bisector Theorem

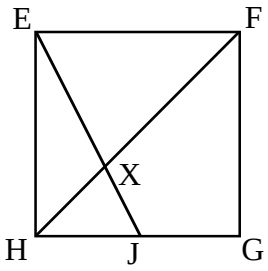
Geometry

Find the missing lengths in each diagram.
Round decimal answers to the hundredth or leave them in radical form. Diagrams not to scale.

7. In triangle ABC, AB=14, BC=15, and AC=16. Point X is on BC such that AX bisects angle BAC. Find the length of BX.

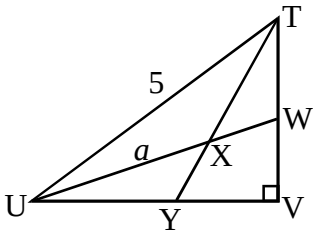


8. Square EFGH has side length 6cm. J is the midpoint of side GH. If FH and EJ intersect at X, what is the length of JX?

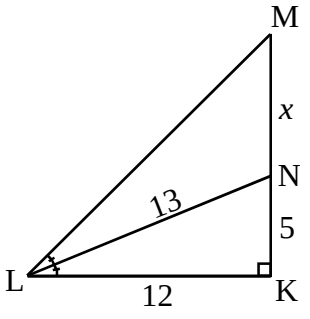


9. In right triangle TUV, TV=3cm and UV=4cm. Angle bisectors UW and TY intersect at X. Find the length of segment UX.

a. Find WV and WT.
b. Find UW (in radical form).
c. Find UX.



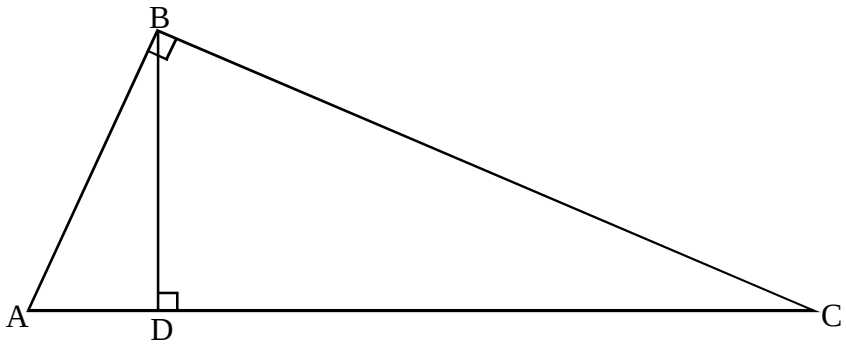
Challenge. The hypotenuse of a 5-12-13 right triangle is the angle bisector of angle M in triangle KLM. Find the length of MN. Express your answer as an improper fraction in simplest form. (If you do this correctly, the denominator will be 119).



Right Triangles

Geometry 11.4

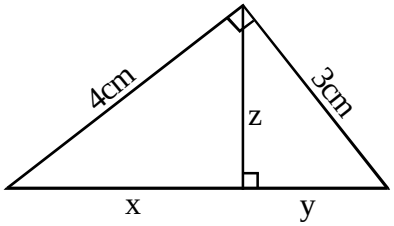
Altitude to the hypotenuse of a right triangle:

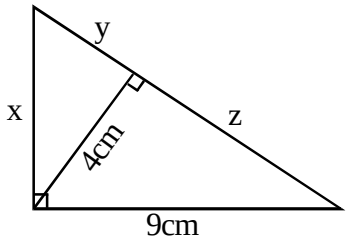


Name three similar triangles in the figure above.

Examples:

Find the missing lengths in the figures below.
Round decimal answers to the hundredth.

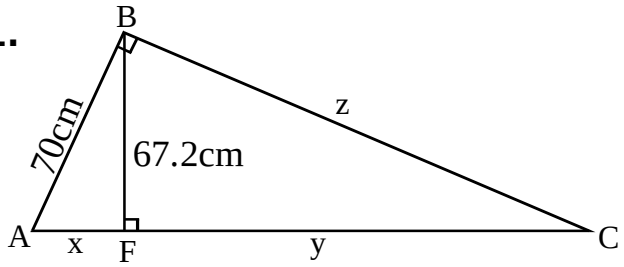




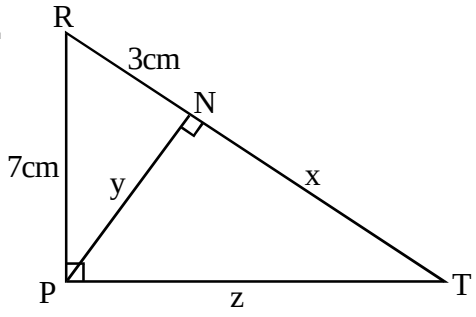
Practice:

Find the missing lengths in the figures below. Round to the hundredth.

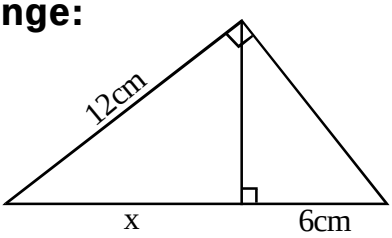
1.



2.



Challenge:

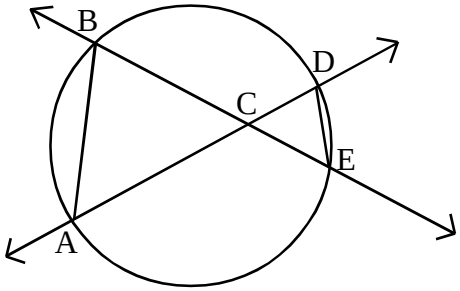


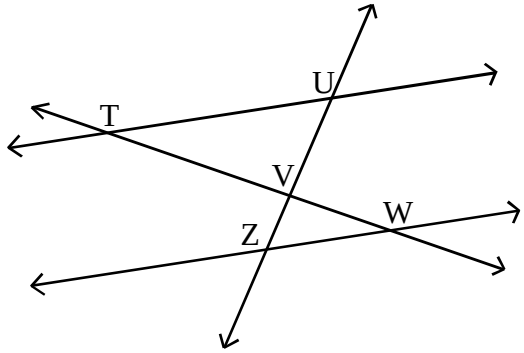
Other Useful Similar Triangles

Geometry 11.4

Two more common types of similar triangles:

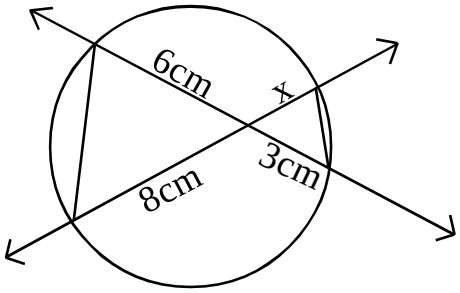
Name the pairs of similar triangles in the diagram below.
Order is VERY important here. Look at which angles are congruent.

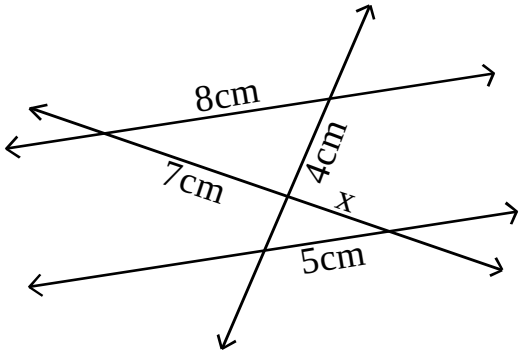




Use what you know to find the missing length 'x' below:

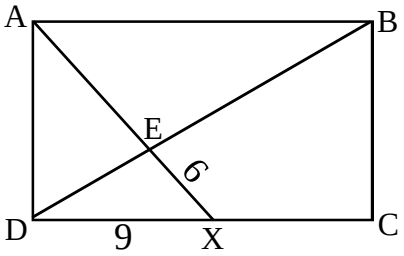
Leave answers as fractions in simplest form.



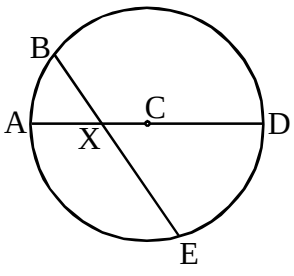


More Difficult Practice: Solve.

1. In rectangle ABCD, X is the midpoint of CD.
   a. Find the length of BD.
   b. Find DE and BE.



2. The diameter of circle C is 22.
   BX=8 and XE=12. Find AX.

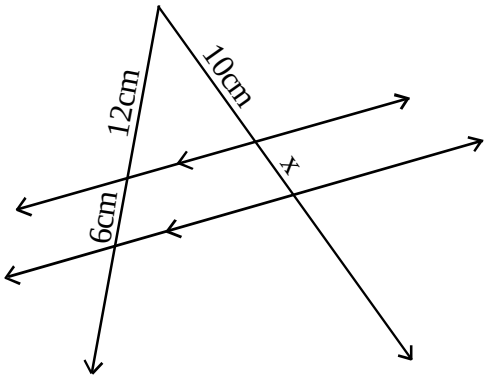


Parallel Lines: Similarity

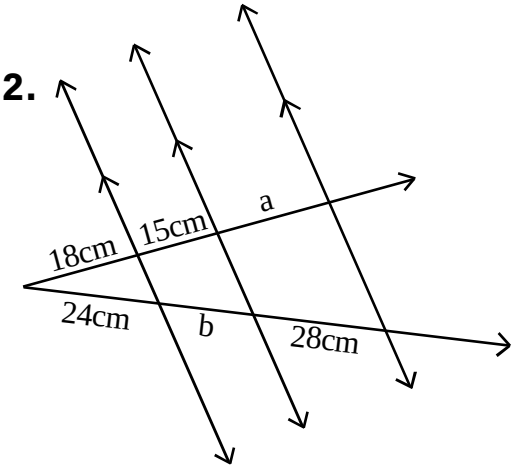
Geometry 11.6

Solve:
Find the missing lengths below.

1.

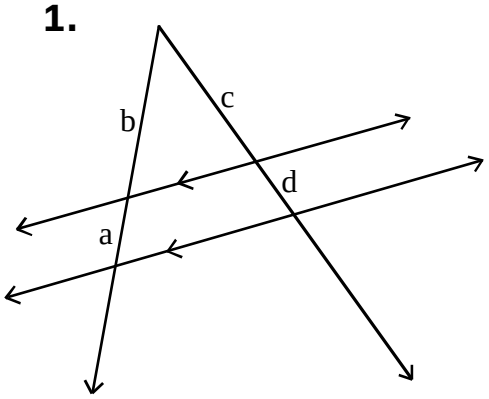


2.

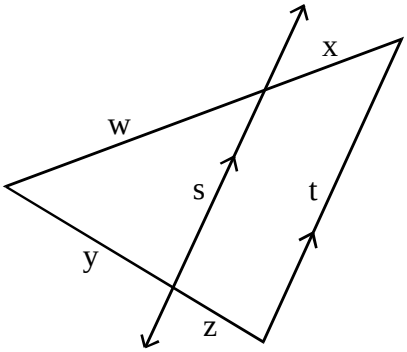


Solve:
Write as many proportions as possible which relate the lengths of sides listed below:

1.



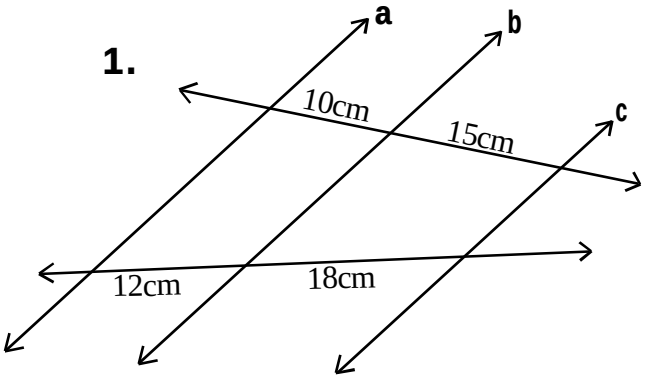
2.



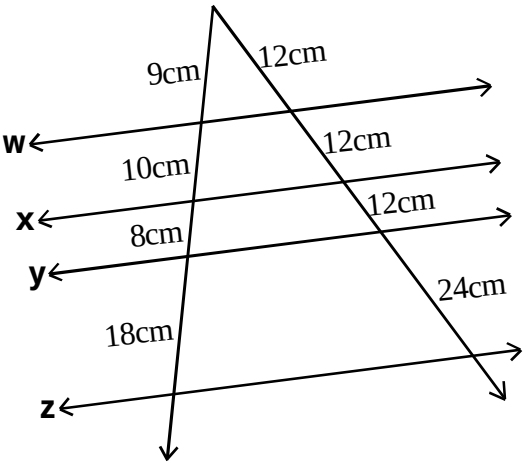
The converse also works. If corresponding lengths between lines are proportional, the lines are parallel.

Solve:
Which of the lines below are parallel?

1.



2.



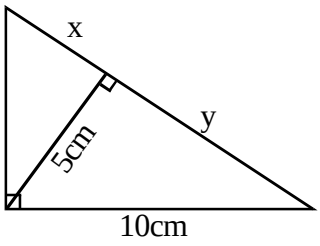
Name_____ Period _____

Similarity in Right Triangles

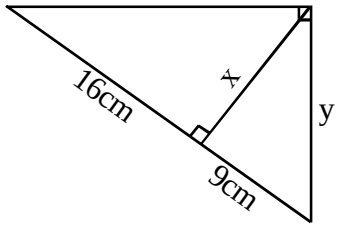
Geometry 11.3

Find the missing lengths in each diagram.
Round decimal answers to the hundredth or leave them in radical form. Diagrams not to scale.

1.



2.

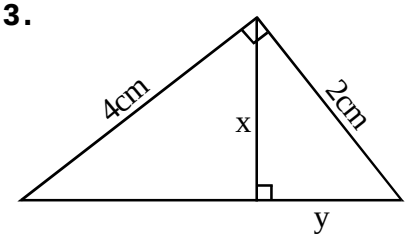


Leave #1 in radical form.

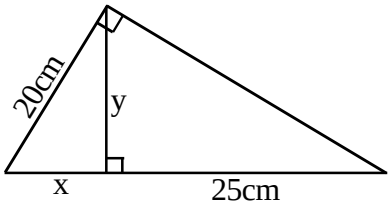
1. x _____ y _____

2. x _____ y _____

3.



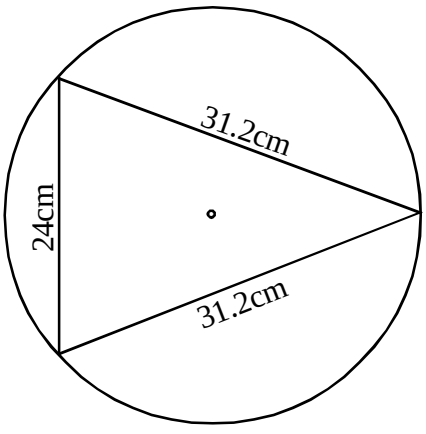
4. (hard: Use Quadratic Formula)



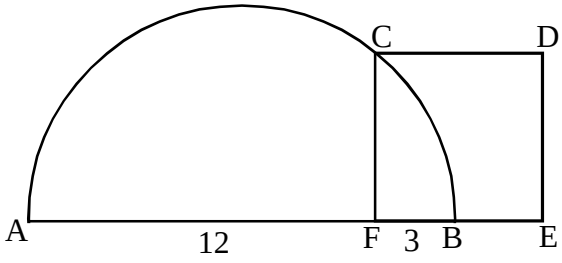
3. x _____ y _____

4. x _____ y _____

5. Find the diameter of the circle:



6. Find the area of square CDEF.



5. diameter = _____

6. area = _____

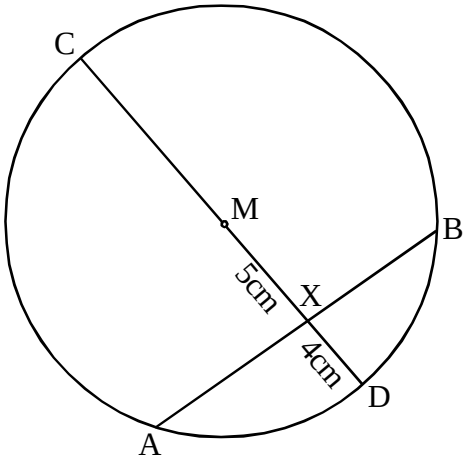
Name_____ Period _____

Similarity

Geometry 11.3

Solve.
Diagrams not to scale.

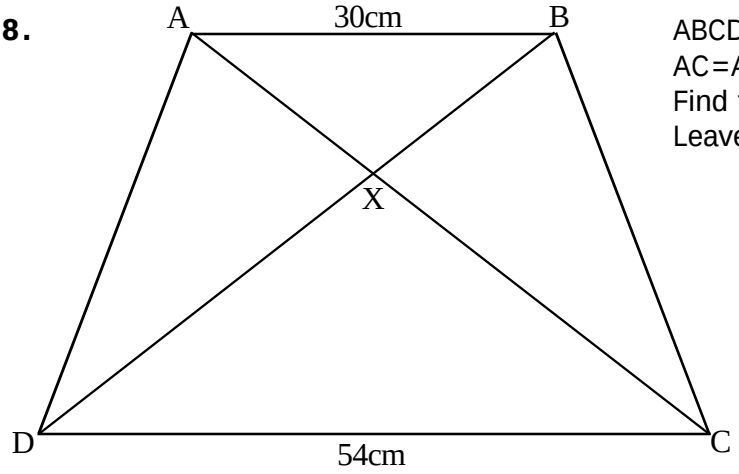
7.



CD is a diameter.
AB=15
Find AX (AX>BX)

7. _____

8.



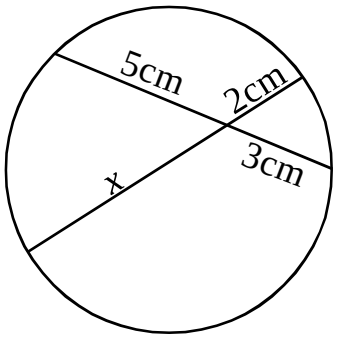
ABCD is an isosceles trapezoid.
AC=AB=56cm
Find the area of triangle ABX.
Leave your answer in simplifed radical form.

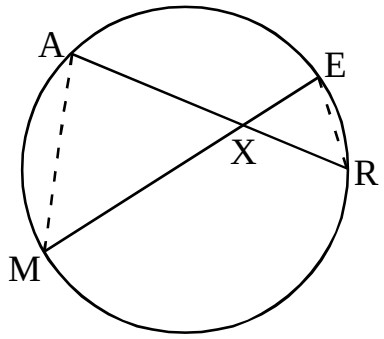
8. _____

Power of a Point:

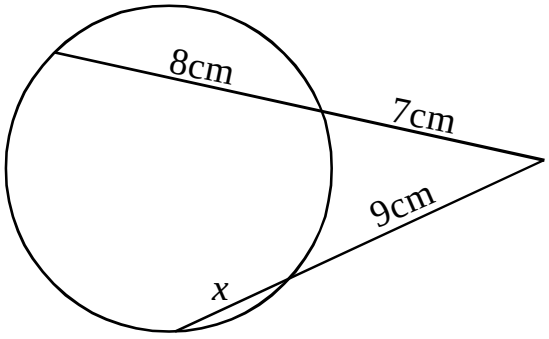
Geometry

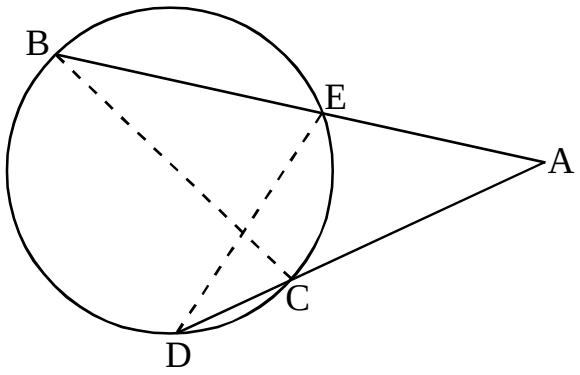
Case 1: Two chords. Find the missing length x:



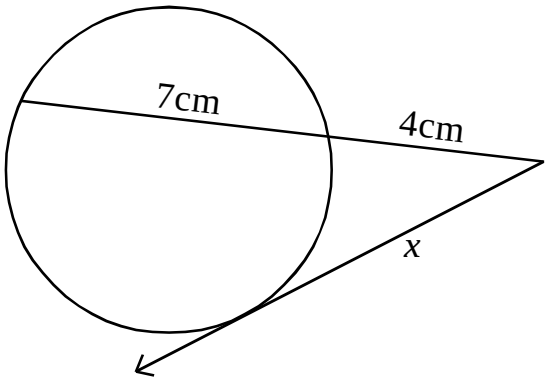


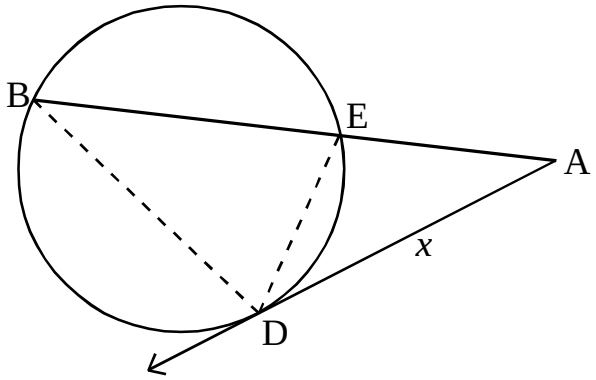
Case 2: Two secants. Find the missing length x:



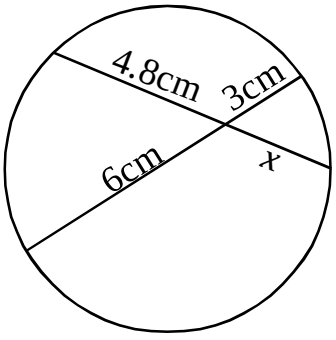


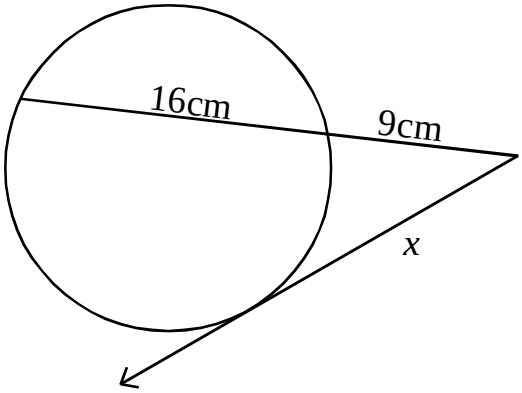
Case 3: Tangent/Secant. Find the missing length x:

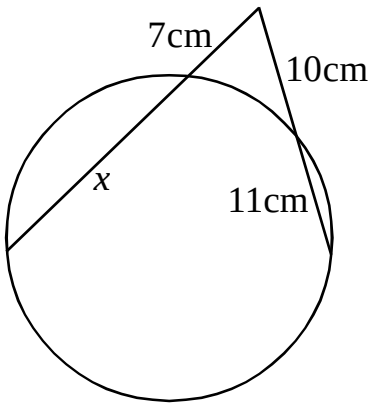




Practice: Find the missing length (not to scale):







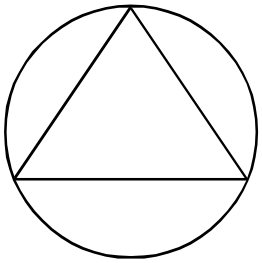
Name_____ Period _____

Similarity

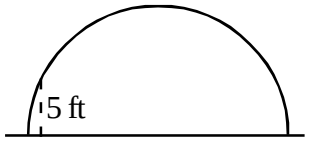
Geometry

Solve.
Diagrams not to scale.

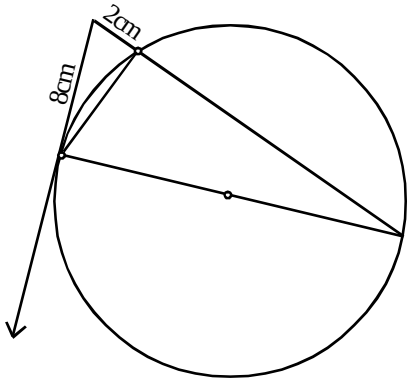
1. An isosceles triangle with congruent side length of 5 and base 6 is inscribed within a circle. What is the diameter of the circle?



2. You are estimating the height of a semicircular arch. Exactly 1 foot from the base of the arch, the arch is 5 feet tall. What is the height of the arch at its center?



3. What is the area of the circle below (in terms of Pi)?



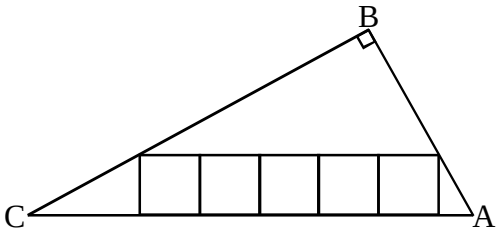
Name_____ Period _____

Similarity

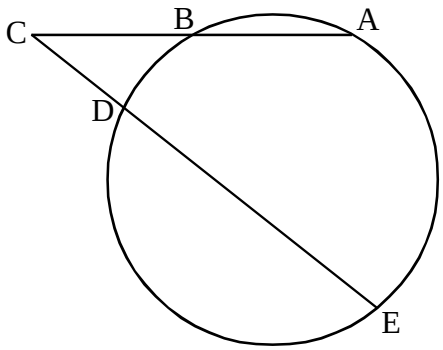
Geometry

Solve.
Diagrams not to scale.

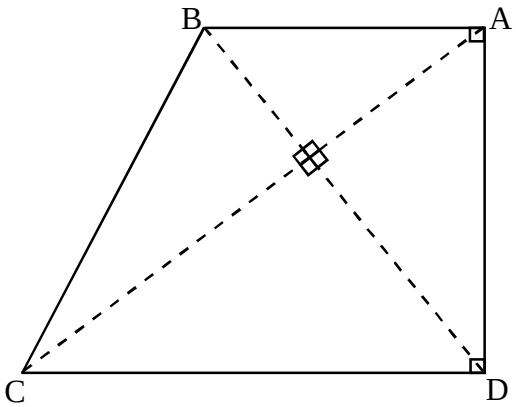
4. What is the combined area of the five squares below if AB=3cm and BC=6cm?



5. Find the length of DE in the diagram below if AB = BC = 3 and CD = 2.



6. What is the area of the right trapezoid ABCD below if AB = 8, and CD = 12?



Area and Volume: Similarity

Geometry 11.5

Changing dimensions of similar figures.
Examples:

1. A square has 3-inch sides. A larger square has 6-inch sides. How many times larger is the area of the large square than the smaller one?

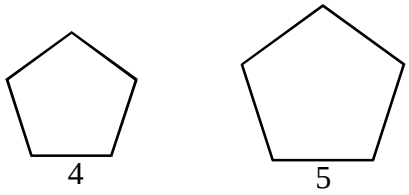
2. A cube has 3-inch sides. A larger cube has 6-inch sides. How many times larger is the volume of the large cube than the smaller one?

3. A circle has a 3-inch radius. A larger circle's radius is five times as long. How many times greater is its area of the large circle than the small one?

4. The volume of a cylinder is $15cm^3$. What would be the volume of a cylinder with 3 times the radius and height?

Similar figures have the same shape and different size. For example, right cones can be similar if their heights and radii are proportional.

The ratio of the lengths of sides in simlar figures is called the **scale factor,** for example the scale factor for the pentagons below is 1.25



If similar two-dimensional figures are increased (or decreased) by a scale factor of n, then the resulting area will change by a factor of n^2.

If similar three-dimensional figures are increased (or decreased) by a scale factor of n, then the resulting volume will change by a factor of n^3.

Solve:

1. A regular pentagon with 5-inch sides has an area of approximately 27.5 square inches. What would the area of a regular pentagon with 10-inch sides be? What about a regular pentagon with 7-inch sides?

2. Similar cones are 3-inches tall and 5-inches tall. What is the ratio of the volume of the larger cone to the volume of the smaller one?

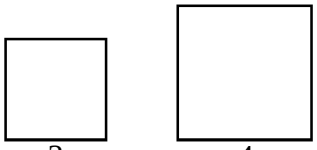
Name_____ Period _____

Area and Volume: Similarity

Geometry 11.5

Solve:

1. What is the ratio of the area of the square on the right to the area of the square on the left? Write your answer as both an improper fraction and a decimal.



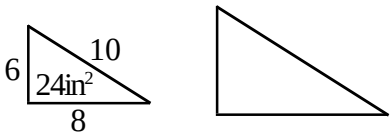
1. _____ or _____

2. Squares have perimeters of 12 and 18 inches. What is the ratio of the area of the larger square to the area of the smaller one?

2. _____

3. Similar triangles have perimeters of 24 and 36 inches. The area of the smaller triangle is $24in^2$. What is the area of the larger triangle?

hint: It is NOT $36in^2$. Use the scale factor. Only use the diagram below if you have to.



3. _____

4. Similar polygons have areas of $18in^2$ and $8in^2$. The larger polygon has a perimeter of 30 inches. What is the perimeter of the smaller polygon?

hint: use the scale factor of their areas to find the scale factor of their perimeters.

4. _____

5. A cube has edge length 4. The edge length is increased to 10.

5a. By what factor has the surface area increased?

5a. _____

5b. By what factor has the volume increased?

5b. _____

6. Cube A has edge length 3. What is the edge length of cube B if it has twice the volume of cube A (to the hundredth).

6. _____

7. Tetrahedron A has edge length 5. What is the edge length of tetrahedron B if it has twice the volume of tetrahedron A (to the hundredth).

7. _____

Name_____ Period _____

Area and Volume: Similarity

Geometry 11.5

With similar figures A and B, if you know the ratio of the side length a/b then the ratio of their areas is $(a/b)^2$ and the ratio of their volumes (if applicable) is $(a/b)^3$.

	Sides/ Perimeter	Area	Volume
Ratio:	$\frac{a}{b}$	$\left(\frac{a}{b}\right)^2$	$\left(\frac{a}{b}\right)^3$

Example:
The edges of a regular tetrahedron are 6cm long. The surface area is about 62cm^2 and the volume is about 25cm^3. Approximate the surface area and volume of a tetrahedron whose edges are 7cm long.

Area = $\left(\frac{7}{6}\right)^2 = \frac{49}{36}(62) \approx 84.4cm^2$ Volume = $\left(\frac{7}{6}\right)^3 = \frac{343}{216}(25) \approx 39.7cm^2$

Practice:

1. The area of a circle is 5cm^2. If the radius is tripled, what will be the area of the new circle?

1. _____

2. The perimeter of a regular heptagon is 19 inches and its area is approximately 27 square inches. Approximate the area of a regular heptagon whose perimeter is 24 inches. (to the tenth)

2. _____

3. The volume of a sphere whose radius is 1.85cm is about 26.52cm^3. Approximate the volume of a sphere whose radius is twice the original (3.7cm). Check your answer using the volume formula for a sphere.

3. _____

4. A regular octahedron has 12cm edges. Its volume is approximately 815cm^3, and its surface area is about 499cm^2. Approximate the surface area and volume of a reguar octahedron with 8cm edges to the nearest whole number.

4. Vol. _____ S.A._____

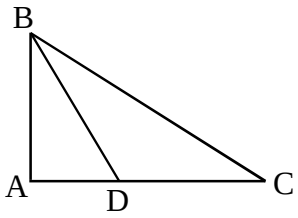
Name_____ Period _____

Area and Volume: Similarity

Geometry 11.5

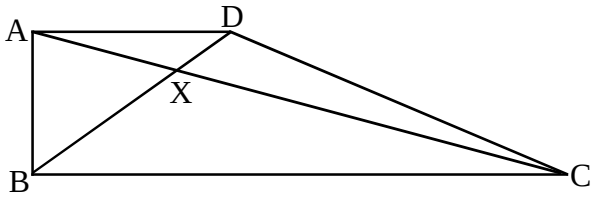
Mixed Review: Solve. Round decimal answers to the hundredth.

5. In right triangle ABC, AB=3 and BC=5. Angle B is bisected by BD, with D on AC. Find the length of angle bisector BD.



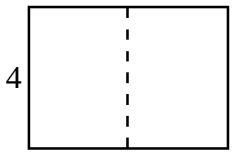
5. _____

6. In right trapezoid ABCD, diagonals AC and BD intersect at X. AB=12, BD=20, and AC=37. Find the length of AX to the nearest hundredth.



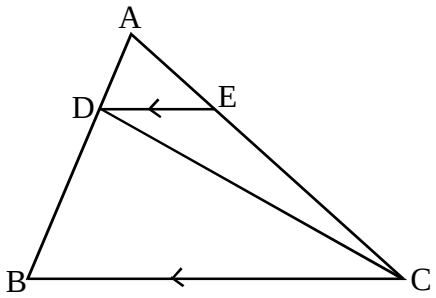
6. _____

7. When cut in half, a sheet of paper yields two smaller rectangular sheets which are similar to the original sheet. If the original sheet is 4 inches tall, how wide is it? (leave in radical form)



7. _____

8. In the triangle below, BC is four times the length of DE.
If the area of triangle ADE is 20cm^2, what is the area of triangle DCE?



8. _____

9. The volume of a regular icosahedron with 5-inch edges is about 270in^3. What is the edge length of a regular icosahedron whose volume is 10in^3?

9. _____

Name_____ Period _____

Area and Volume: Similarity

Geometry 11.5

Using scale factor and similarity:

Examples:

The area of a triangle with side lengths 5, 7, and 10 is $2\sqrt{66}$. What would be the area of a triangle with side lengths of 50, 70, and 100?

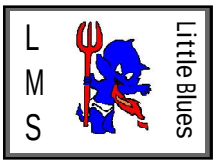
_____

Mr. Batterson has his own action figure. It looks exactly like him, but it is just 11.5 inches tall. Mr. Batterson is 5'-9" and weighs 162 pounds. If the action figure is the same density as Mr. Batterson, how much does it weigh?

_____

Practice:

1. You are painting a mural onto your school based on a drawing you created on a sheet of paper that is 8.5 x 11 inches. The mural will be 34 by 44 feet. The mascot portion of the mural is $10in^2$ on the sheet of paper. What will the area of the mascot portion be on the actual building in square feet? (hint: the answer is between 100 and 200 ft^2)



1. _____

2. An artist carves three perfectly similar cones out of marble. The smaller of the three cones is 3 feet tall and weighs exactly one ton (2000 lbs)! The other pyramids are 40 inches and 44 inches tall. How much more does the largest cone weigh than the smallest cone (to the pound)?

2. _____

Name_____ Period _____

Area and Volume: Similarity

Geometry 11.5

Practice:

3. A restaurant in Italy is trying to break the world record for the largest pizza ever baked. To break the record, they need to bake a pizza with a 126 foot diameter. Their normal pizzas have only a 14 inch diameter and it takes a pound of cheese to cover the pizza. How many pounds of cheese will they need to cover the giant pizza?

3. _____

4. Your mom has a garden statue that is an exact replica of the statues at Easter Island. He garden statue is just 2 feet tall. The real statues average 14 feet tall and weigh 27,440 pounds. If her stone replica is made of the same stone as the real "moai" statues, how much does it weigh (in pounds)?

4. _____

5. Brass "Beast" trophies are handed out for Mr. Batterson's math class. Each is solid brass and the third place trophy is 15 inches tall. Mr. B. wants the first place trophy to be exactly three times as large (by mass) as the 3rd place trophy. To the tenth of an inch, how tall should it be?

5. _____

6. The surface area of a solid clay hemisphere is $10cm^2$. A larger solid clay hemisphere has a surface area of $40cm^2$. If the larger hemisphere weighs 2 pounds, how much does the smaller one weigh?

5. _____

Name_____ Period _____

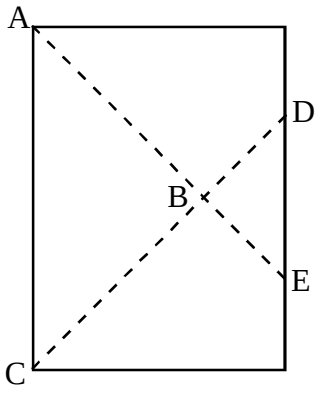
Similarity Practice

Geometry 11.7

Solve:

Each of the problems below can be solved using proportional reasoning. Working with a partner, solve as many as you can.

1. The top right corner of a 8.5 x 11 sheet of paper is folded down and left to align with the left edge,and the bottom right corner is folded up and left so that the fold lines look like the diagram below. What is the ratio of the area of the large isosceles triangle ABC to the smaller triangle DBE?



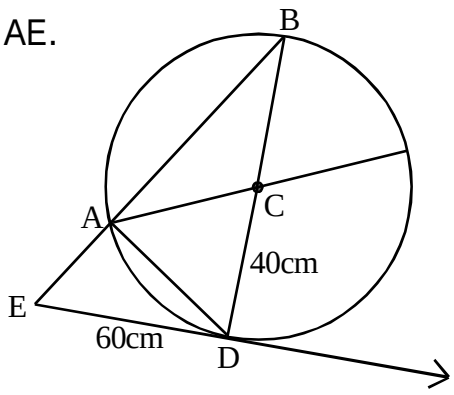
2. AB and CD are chords of a circle which intersect at X. If AX = 3, BX = 4, and CX = 2 what is the length of DX?

3. Find the length of chords AB, AD, and AE.

AB _____

AD _____

AE _____



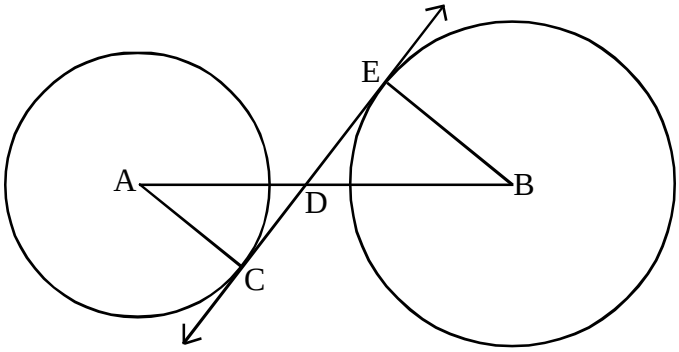
Name_____ Period _____

Similarity Practice

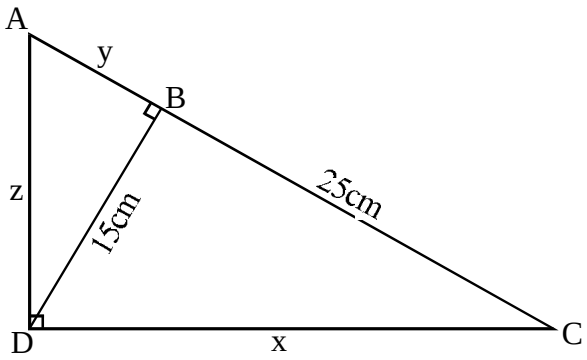
Geometry 11.7

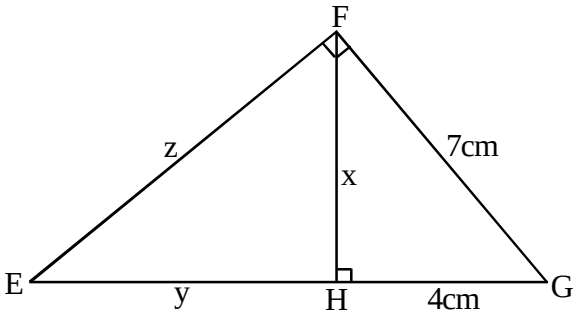
Solve:

4. Circle A has a radius of 28 inches, and circle B has a 44-inch radius. The distance between their centers is 90 inches. What is the length of segment CE, which is externally tangent to both circles?

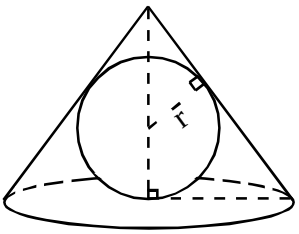


5. In each right triangle below, solve for lengths x, y, and z. Leave answers in radical form or round to the hundredth.





6. A sphere is inscribed in a cone whose radius is 6cm and whose height is 8cm. Find the volume of the sphere.



Name_____ Period _____

Area and Volume: Similarity

Geometry

Practice:

1. The edge length of a cube is doubled. The original cube had a surface area of 96cm^2. What is the surface area of the larger cube?

1. _____

2. The edge length of a triangular prism is doubled. The original prism had a volume of 10cm^3. What is the volume of the larger prism?

2. _____

3. A 10x10x20 rectangular prism has each of its edge lengths increased by 40%. By what percent is its volume increased?

3. _____

4. Similar cones have volumes of 10cm^3 and 80cm^3. If the height of the smaller cone is 10cm, what is the height of the larger cone?

4. _____

5. An icosahedron has each of its edge lengths increased by 30%. By what percent is its volume increased?

5. _____

6. Similar pyramids have surface areas of 50cm^2 and 80cm^2. What is the ratio of the volume of the smaller pyramid to the volume of the larger one? Express your answer as a decimal to the nearest hundredth.

6. _____

7. A cube has an interior diagonal length of $5\sqrt{3}$ cm. A larger cube has twice the surface area. What is the length of its interior diagonal?

7. _____

8. Michael has a scale model of a Ford Mustang. It takes a proportional amount of paint to coat the model as the real mustang. If the model is 1/12 the length of a real Mustang and requires 2 ounces to paint, how many gallons will it take to paint a real Mustang (128oz = 1 gal.)

8. _____

Name_____ Period _____

Similarity Practice Test

Geometry 11.7

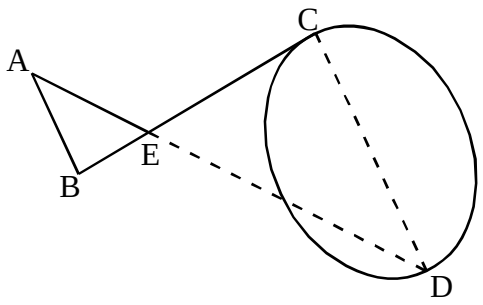
Solve:

1. At 3pm, Sam's shadow is 6 inches taller than Mary's, even though he is only 4 inches taller than she is. If Sam is 9 inches taller than Allison, how much longer is Sam's shadow than Allison's?

1. _____

2. To determine the width of a lake, the measurements below are taken. What is the width CD of the lake? (note: AB||CD)

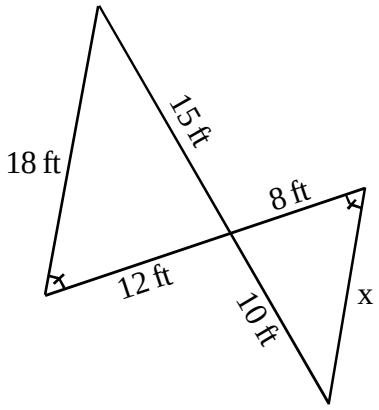
AB = 32 feet.
BE = 22.4 feet
EC = 42 feet



2. _____

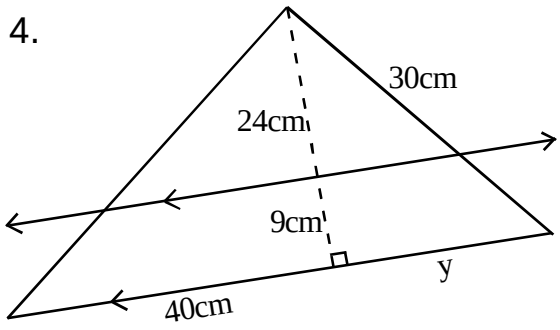
Solve: Find the missing lengths below.

3.



3. x = _____

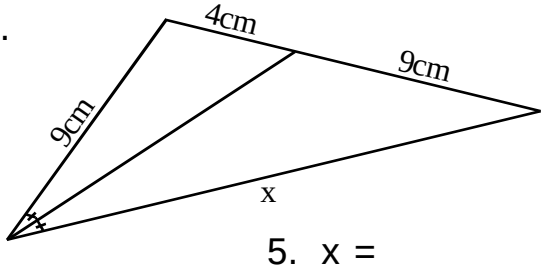
4.



4. y = _____

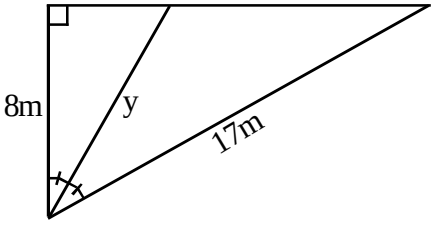
Solve: Find the missing lengths below. Round to the <u>hundredth</u>.

5.



5. x = _____

6.



6. y = _____

Name_____ Period _____

Similarity Practice Test

Geometry 11.7

Solve: Do not round your answers.

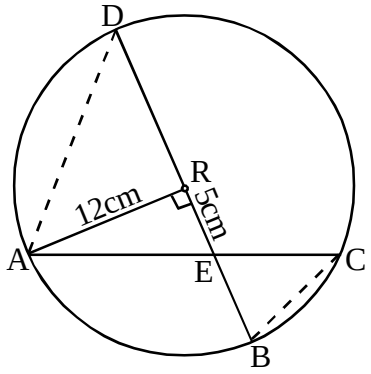
7. Similar pyramids have altitudes of 3cm and 4cm. The volume of the smaller pyramid is $30cm^3$. What is the volume of the larger pyramid?

7. _____

8. At an art museum, there is a 14-foot tall solid marble statue of a horse. At the gift shop you buy a solid marble replica of the statue that weighs about 2 pounds and is only 8 inches tall. Estimate the weight of the full sized original statue.

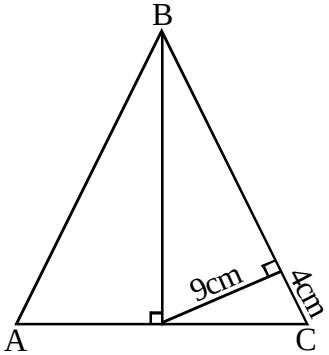
8. _____

9. Find the length of chord $\overline{AC}$ in circle R below.
(Dotted lines are hints, fill-in everything you know and you'll get there.)



9. _____

10. What is the area of isosceles triangle ABC below to the hundredth?



10. _____

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