Transformations

A transformation is a change in coordinates plotted on the plane.
We will learn about four types of transformations on the plane:
Translations, Reflections, Rotations, and Dilations.

Translations simply move the coordinates of the figure and can be represented by coordinate rules:
Begin with the first graph on your sheet.
Plot the following coordinates and connect them to form a triangle:
(2,1) (2,5) (8,2)

1. Transform the figure using the rule: **(x', y') = (x-4, y-7)**
2. Transform the original figure using the rule: **(x-8, y+1)**
3. Write the rule would you use to create the following translation:
(12,-6) (12,-2) (18,-5)

Reflections mirror the coordinates across a line or axis:
New graph:
Plot the following coordinates and connect them to form a triangle:
(4,2) (6,4) (7,2)

1. Draw a reflection of the figure across the x-axis. What are the new coordinates? _______________
2. Draw a reflection (of the original) across the y-axis. What are the new coordinates? _______________
3. Draw the line y=x. reflect the figure across y=x. What are the new coordinates? _______________

Write three rules based on what you figured out above:

To reflect across the x-axis ___________________________. (x,y)→(__,__)

To reflect across the y-axis ___________________________. (x,y)→(__,__)

To reflect across y=x ________________________________. (x,y)→(__,__)

Transformations

Rotations actually act as double reflections:
New Graph
Plot the following coordinates and connect them to form a triangle:
(3,2) (6,5) (7,3)
Use the RULES FOR REFLECTIONS to get the coordinates below.
1. Reflect the coordinates across the x-axis , and then reflect THE NEW COORDINATES across the y-axis. This is a 180^o rotation.
New coordinates:______________
Write the rule: __________________________________ (x,y)→(__,__)

2. Reflect the coordinates across the y=x , and then reflect THE NEW COORDINATES across the x-axis. This is a 90^o Clockwise rotation.
New coordinates:______________
Write the rule: __________________________________ (x,y)→(__,__)

3. Reflect the coordinates across the y=x , and then reflect THE NEW COORDINATES across the y-axis. This is a 90^o Counter-Clockwise rotation.
New coordinates:______________
Write the rule: __________________________________ (x,y)→(__,__)

Rules: 180^o: negate x and y.
90^o CW: switch the coordinates, negate y.
90^o CCW: switch the coordinates, negate x.

When you get confused (and you will), try rotating a point like (1,2) to write the rule. It is easy enough to do even without a graph.

Practice: Try to complete each without looking at your written rules.
1. The coordinates (9,5) (3,-4) (-9,-3) and (-1,4) form a quadrilateral.
- **a.** What would be the coordinates for a 90^o CW rotation?
- **b.** What would be the coordinates for a 270^o CW rotation?
- **c.** What would be the coordinates for a 180^o rotation?
- **d.** Plot all three rotations (careful!)

Dilations are enlargements or reductions and involve multiplication by a scale factor.
New Graph
Plot the following coordinates and connect them to form a triangle:
(-2,2) (3,1) (-1,-2)
Multipy all three coordinates by 3 and plot the result.

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Transformations Practice

Geometry

Practice:
Use the following points for #1-8:
Parallelogram ABCD:
A(1,-3) **B**(2,-6) **C**(-5,-6) **D**(-6,-3)

1. What coordinates would be used to move the figure two units down and four units to the right?

____

2. What coordinates would be used to reflect the figure across the x-axis?

____

3. What coordinates would be used to reflect the coordinates across the y-axis?

____

4. What transformation occurs when you reflect across both axis? (list coordinates and describe the transformation):

____

5. Write the coordinates for a 90° clockwise rotation about the origin.

____

6. Write the coordinate rule for a translation two units down followed by a reflection across the x-axis, then list the coordinates:

(x,y)→ (_____,_____) _________________________________

7. Write the coordinate rule for a 180° rotation followed by a translation 3 units down and four units to the left, then list the coordinates.

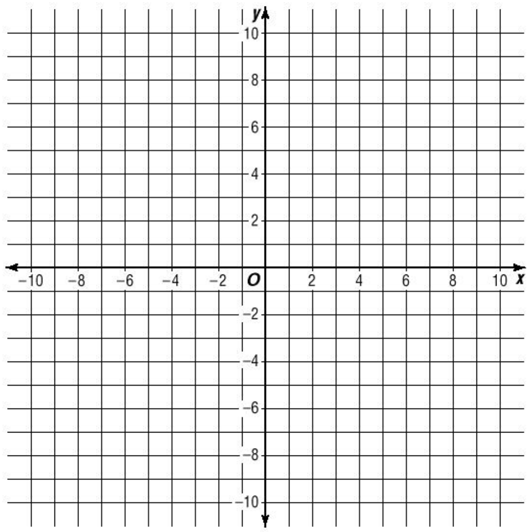
(x,y)→ (_____,_____) _________________________________

8. Write the coordinate rule that would be used to reflect the coordinates across y=-x, then list the new coordinates. (This one may be easier to graph first.)

(x,y) (_____,_____) _________________________________

On the back of this sheet, plot the original coordinates and the transformation for each number listed on the numbered graphs:
#1, 2, 5, 6, 7, 8.

1.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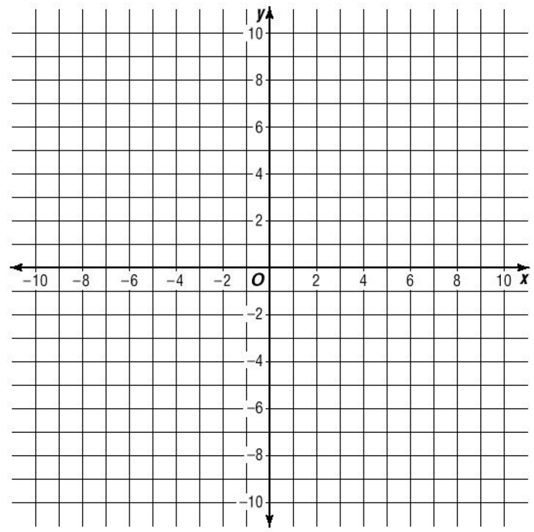
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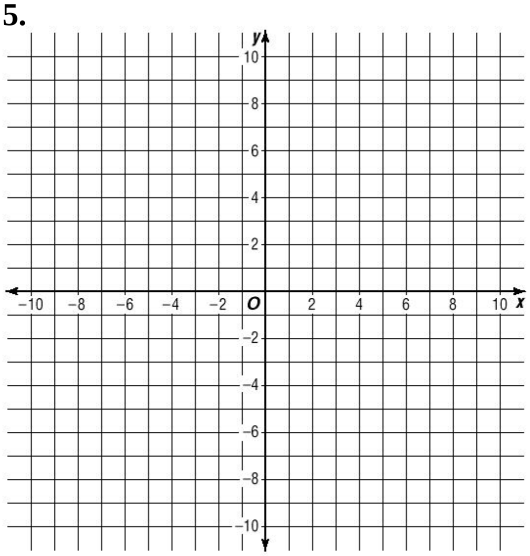
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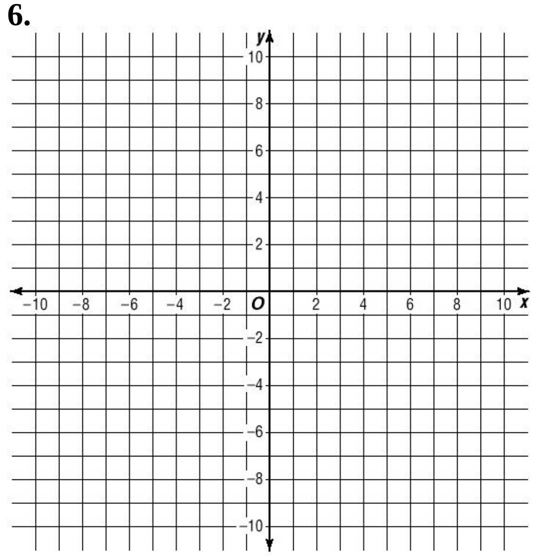
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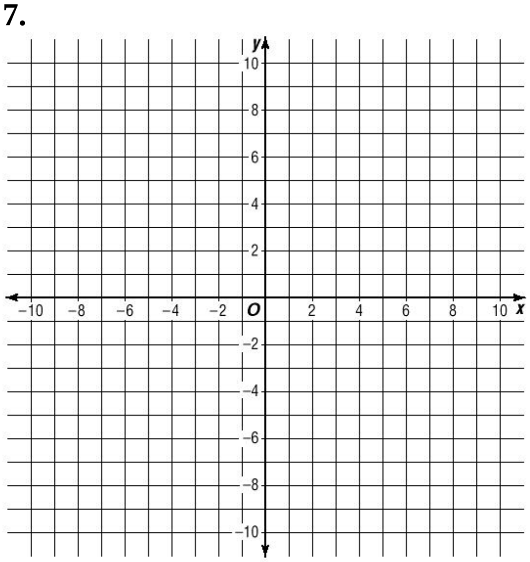
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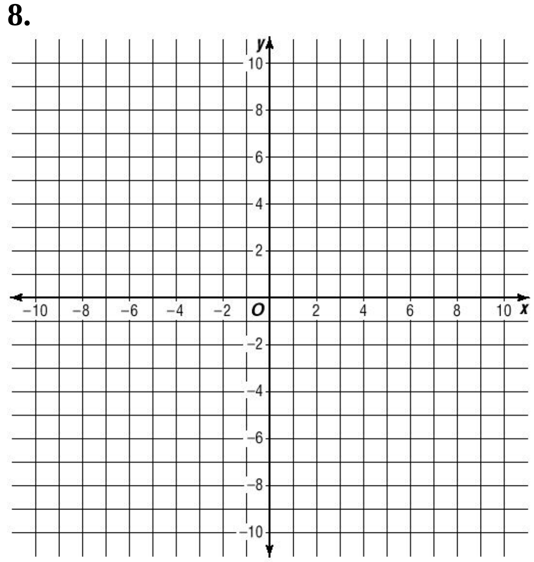
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Transformations w/ Matrices

Geometry 14.1

Matrices can be used to represent coordinate pairs or sets of points on the plane.

Ex. A(-3, 7) B(4, 5) C(-2, 9)

$$\begin{array}{c} \\ X \\ Y \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} -3 & 4 & -2 \\ 7 & 5 & 9 \end{bmatrix} \end{array}$$

Matrix Addition: Matrices must have the same number of rows and columns.

$$\begin{bmatrix} -3 & 4 & -2 \\ 7 & 5 & 9 \end{bmatrix} + \begin{bmatrix} -1 & 0 & -8 \\ -3 & -2 & 5 \end{bmatrix} = \begin{bmatrix} -4 & 4 & -10 \\ 4 & 3 & 14 \end{bmatrix}$$

Translations: How could you display the translation of the coordinates listed below up 5 units and left 3 units using matrix addition?

A(-4, 5) **B**(-2, 1) **C**(9,0) **D**(2, -3)

Scalar Multiplication with Matrices: Simply multiply the scalar by the coordinates.

$$-5\begin{bmatrix} -3 & 4 & -2 \\ 7 & 5 & 9 \end{bmatrix} = \begin{bmatrix} 15 & -20 & 10 \\ -35 & -25 & -45 \end{bmatrix}$$

Dilations: How could you represent the dilation of the points below using scalar multiplication by scale factor of 2:

A(2, -1) **B**(-3, 5) **C**(9,-2) **D**(4, 6)

180^o rotation: What scalar could you use to create a 180^o rotation of coordinates?

Practice:

Graph the following coordinates and use scalar multiplication to create a 180^o rotation and a dilation with a scale factor of 3 (then graph it).

A(-3, 2) **B**(-1, -2) **C**(2,3)

Transformations w/ Matrices

Matrix multiplication is MUCH more tedious and difficult, but fortunately in geometry the skill is limited to basic matrices. The number of columns in the first matrix must match the number of rows in the second matrix.
Multiply the rows of the first by the columns of the second.

$$\begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix} \begin{bmatrix} -5 & 4 \\ -3 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1(-5)+2(-3)+(-3)(-1) & -1(4)+2(2)+(-3)0 \\ 4(-5)+(-5)(-3)+6(-1) & 4(4)+(-5)2+6(0) \end{bmatrix}$$

Final answer: $\begin{bmatrix} 2 & 0 \\ -11 & 6 \end{bmatrix}$

Try this one on your own: The rows and columns are labeled to help you know how to begin:

$$\begin{matrix} X \\ Y \\ Z \end{matrix} \begin{bmatrix} -5 & 4 \\ -3 & 2 \\ -1 & 0 \end{bmatrix} \overset{\begin{matrix} A & \quad B & \quad C \end{matrix}}{\begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}} = \begin{bmatrix} XA & XB & XC \\ YA & YB & YC \\ ZA & ZB & ZC \end{bmatrix}$$

How does this relate to transformations?
Lucky for us... someone found a way to apply this to transformations (reflections specifically) on the plane, and the state of NC decided it would be a good thing to include in its geometry curriculum.

Take a look at what happens when we multiply the matrix below by the set of points: (-1,4) (2, -5) (-3,6)

$$\begin{matrix} Y \\ Z \end{matrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \overset{\begin{matrix} A & \quad B & \quad C \end{matrix}}{\begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}} = \begin{bmatrix} YA & YB & YC \\ ZA & ZB & ZC \end{bmatrix} =$$

What transformation occurs?

$$\begin{matrix} Y \\ Z \end{matrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \overset{\begin{matrix} A & \quad B & \quad C \end{matrix}}{\begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}} = \begin{bmatrix} YA & YB & YC \\ ZA & ZB & ZC \end{bmatrix} =$$

What transformation do you think this creates?

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Matrix Practice

Geometry 14.1

Matrix multiplication:
Complete each multiplication problem. Multiply the rows of the first matrix by the columns of the second.

1.

$$\begin{bmatrix} -3 & 2 & -1 \\ 8 & -4 & 7 \end{bmatrix} \begin{bmatrix} -1 & 7 \\ 3 & 0 \\ -5 & 6 \end{bmatrix} =$$

Final answer (2x2): $\begin{bmatrix} & \\ & \end{bmatrix}$

2.

$$\begin{bmatrix} -1 & 7 \\ 3 & 0 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} -3 & 2 & -1 \\ 8 & -4 & 7 \end{bmatrix} =$$

Final answer (3x3): $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

3.

$$\begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -4 & 1 & 0 \\ 3 & -2 & -1 \end{bmatrix} =$$

Final answer (2x3): $\begin{bmatrix} & & \\ & & \end{bmatrix}$

4.

$$\begin{bmatrix} 4 & -1 \\ 9 & 5 \end{bmatrix} \begin{bmatrix} -2 & 4 & -6 \\ 0 & -1 & 3 \end{bmatrix} =$$

Final answer (2x3): $\begin{bmatrix} & & \\ & & \end{bmatrix}$

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Matrix Practice

Geometry 14.1

Matrix multiplication:
Determine what transformation occurs with each multiplication:
Write the algebraic rule for each transformation and describe it.

ex: (x,y)→(-x, -y) 180° rotation

5.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -5 & 0 & -2 \\ 3 & -1 & 4 \end{bmatrix} =$$

____

6.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ -5 & -4 & 5 \end{bmatrix} =$$

____

7.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 & -1 \\ 2 & -3 & 9 \end{bmatrix} =$$

____

8.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & -3 \\ 0 & -6 & 8 \end{bmatrix} =$$

____

9.

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -4 & 5 & -6 \\ 9 & -8 & 7 \end{bmatrix} =$$

____

10. Write a matrix multiplication that would create a dilation with a scale factor of 2 and a reflection across y=x.

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Transformations Review

Geometry 14.1

Each algebraic rule below represents a transformation. Describe each.
Not sure? Graph a couple of sample points.

1. $(x, y) \rightarrow (x+3, y-5)$ ____

2. $(x, y) \rightarrow (-x, y)$ ____

3. $(x, y) \rightarrow (-x, -y)$ ____

4. $(x, y) \rightarrow (-y, -x)$ ____

5. $(x, y) \rightarrow (y, -x)$ ____

6. $(x, y) \rightarrow (-y, x)$ ____

7. $(x, y) \rightarrow (-y-3, x-5)$ ____

8. $(x, y) \rightarrow (-y+2, -x)$ ____

What matrix would be used in multiplication to create each transformation below?

9. A 180° rotation. $\begin{bmatrix} & \\ & \end{bmatrix}$

10. A reflection across x. $\begin{bmatrix} & \\ & \end{bmatrix}$

11. A reflection across y. $\begin{bmatrix} & \\ & \end{bmatrix}$

12. A reflection across y=x. $\begin{bmatrix} & \\ & \end{bmatrix}$

Name________________________ Period _____

Transformations Review

Geometry

Answer each:
What transformation occurs when you do each of the following?

13. Switching the x and y coordinates: ______________________________________

14. Negating the x coordinate(s): ____

15. Adding a positive value to the x coordinate(s): ____________________________

16. Tripling the x and y coordinates: _______________________________________

17. Adding -3 to both the x and y coordinates: ______________________________

18. Multiplying both coordinates by -2: _____________________________________

19. Switching AND negating both coordinates: ______________________________

20. Adding 6 to the x coordinate(s) then negating y. __________________________

Answer each about the matrix multiplication below:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} g & h & j \\ k & m & n \end{bmatrix}$$

21. Values a and c are multiplied by the _______(x or y) coordinates.

22. Values b and d are multiplied by the _______(x or y) coordinates.

23. In this problem, m = ________ + ________.

24. If you only want to create a dilation, which values (a,b,c,d) should be 0? ______

25. In this problem, ______ = 3a + 6b.

Transformations Quiz Review

Geometry 14.1

Describe each algebraic rule below with the transformation it defines.

1. $(x,y) \rightarrow (-x, y)$

2. $(x,y) \rightarrow (-x, -y)$

3. $(x,y) \rightarrow (-y, x)$

4. $(x,y) \rightarrow (y+2, x-5)$

Describe each transformation using an algebraic rule:

1. Reflection across y=-x.
2. 90^{o} rotation clockwise.
3. Reflection across the y-axis followed by a translation up 4, left 5.

Complete the matrix multiplication problem below:

$$\begin{bmatrix} -2 & 5 & -3 \\ 6 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & 7 \\ 8 & 0 \\ -9 & 1 \end{bmatrix} =$$

Complete each matrix multiplication and describe the translation created in each matrix problem below:

1.

$$\begin{bmatrix} -3 & 4 & -2 \\ 7 & 5 & 9 \end{bmatrix} + \begin{bmatrix} -1 & -1 & -1 \\ 4 & 4 & 4 \end{bmatrix} =$$

2.

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 5 & -2 \\ 6 & -4 & 3 \end{bmatrix} =$$

3.

$$\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} -1 & 5 & -2 \\ 6 & -4 & 3 \end{bmatrix} =$$

4.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 5 & -2 \\ 6 & -4 & 3 \end{bmatrix} =$$

What transformation occurs when you:

1. Add positive values to the x and y coordinates.
2. Switch and negate both coordinates.
3. Negate both the x and y coordinates.
4. Negate x, then switch x and y.

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Transformation Practice Quiz

Geometry

Use the following coordinates for 1-8:

A (3, -2) B(0, 4) C(-1, 6)

1. What would the coordinates of A be after a 180° rotation?

1. ____________

2. What would the coordinates of C be after a reflection across the x-axis?

2. ____________

3. The coordinates are changed to A(-2, 3) B(4, 0) and C(6, -1). What transformation occurs?

3. ________________________________

4. The coordinates are changed to (2, 3) (-4, 0) and (-6, -1). What transformation occurs?

4. ________________________________

5. The coordinates are multiplied by the following matrix: What transformation occurs?

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{bmatrix}$$

5. ________________________________

6. What matrix would you multiply by to create a reflection across y=x?

6. $\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$.

7. List the coordinates of B after a translation up 7 units and left 5 units.

7. ____________

8. Write the coordinate rule for a reflection across x followed by a translation down 3 and right 2 units.

8. (x,y)→____________

Name________________________ Period _____

Transformation Practice Quiz

Geometry

Write each coordinate rule described below:

9. 90° rotation clockwise.

9. (x,y)→_____________

10. 90° rotation counter-clockwise.

10. (x,y)→_____________

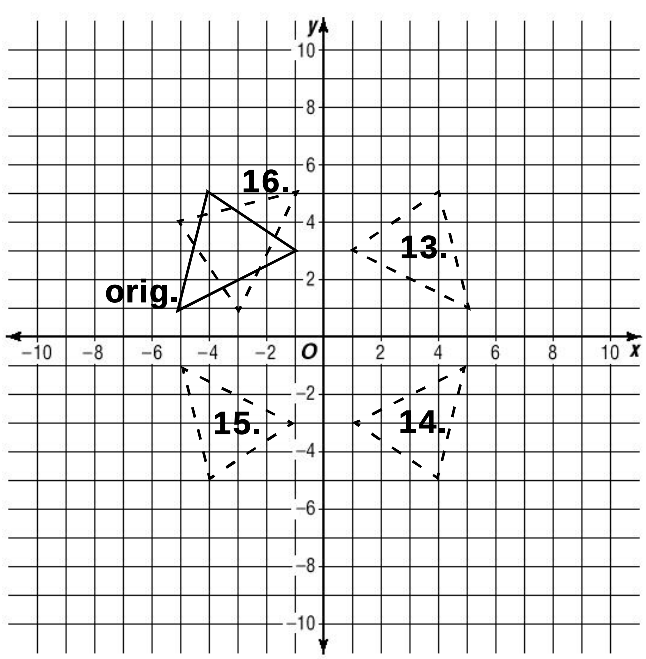
11. Translation up 7 units and left 5 units.

11. (x,y)→_____________

12. Reflection across y=-x.

12. (x,y)→_____________

What matrix would you multiply the original coordinates by to complete each of the following transformations?



13. [ ] .

14. [ ]

15. [ ]

16. [ ]