Inductive Reasoning

Geometry 2.1

Inductive Reasoning:
Observing Patterns to make generalizations is induction.

Example:
Every crow I have seen is black, therefore I generalize that 'all crows are black'. Inferences made by inductive reasoning are not necessarily true, but are supported by evidence.

Think:
How could you use inductive reasoning to find Pi (the ratio of a circle's circumference to its diameter)?

Practice:
Use inductive reasoning to determine the next two numbers in each sequence:

1. 1, 1, 2, 3, 5, 8, 13, ___, ___, ...
2. 5, 4, 6, 3, 7, 2, ___, ___, ...
3. $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \_, \_, ...$
4. 1, 2, 2, 4, 8, ___, ___, ...
5. 2, 3, 5, 7, 11, ___, ___, ...
6. 1, 2, 6, 15, 31, 56, ___, ___, ...

Induction is very important in geometry. Often, a pattern in geometry is recognized before it is fully understood. Today we will use inductive reasoning to discover the sum of the measures of interior angles in a pentagon.

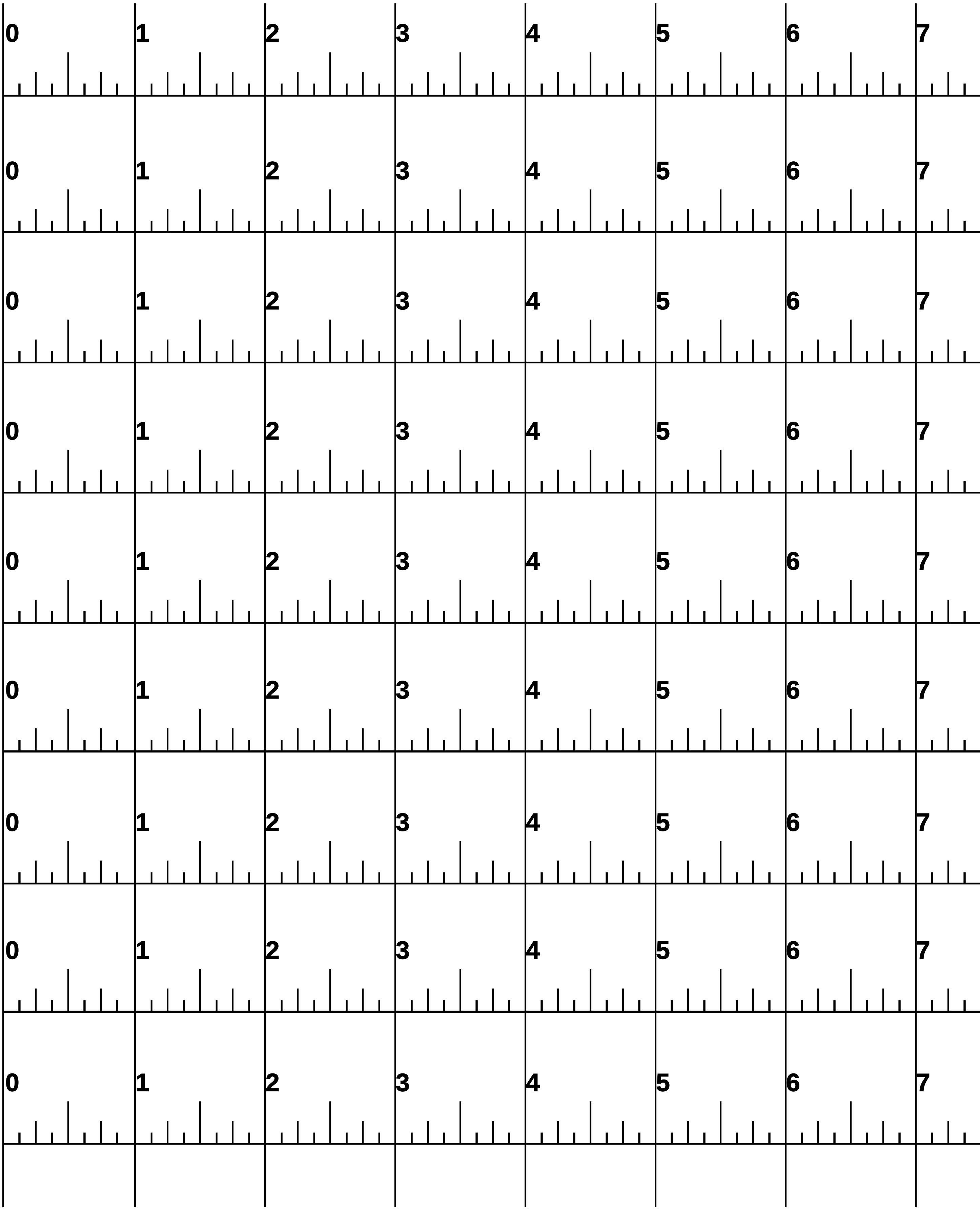
With a partner:
Use a straight-edge to draw an irregular convex pentagon. Trade with a partner, and measure each of the five interior angles in the pentagon. Add these angles and share the data with your partner. We will compare data from other groups in the class to see if we can come up with a generalization.
(If you already know the sum, humor me. It is still a good exercise to demonstrate inductive reasoning).

Now, look for a pattern that will help you discover the sum of the interior angles in figures with more than five sides.
Sum of interior angles in a figure given the number of sides:
3 = 180^o 4=360^o 5 = 540^o 6 = ___^{o} 7 = ___^{o}

Inductive Reasoning

Geometry

Rulers for making circle measurements to find Pi.



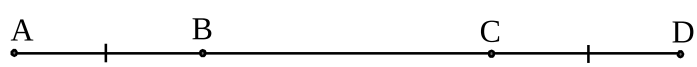
Deductive Reasoning

Deductive Reasoning is the process of using accepted facts and logic to arrive at a valid conclusion.

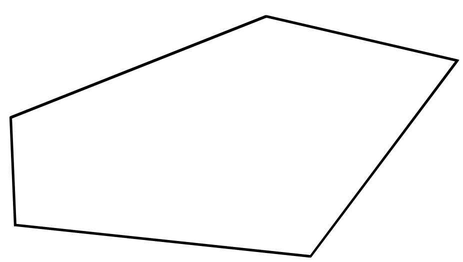
Example:
All mammals have fur (or hair).
Lions are classified as mammals.
Therefore, Lions have fur.

Bad example:
All mammals have fur or hair.
Caterpillars have hair.
Caterpillars are mammals.

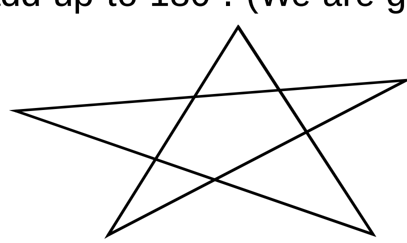
In Geometry:
In the diagram below, what is the relationship between segments AC and BD? Explain why this is true using Algebra.



Applying Deductive Reasoning:
We used inductive reasoning to show that the sum of the interior angles in a pentagon appears to always equal to 540^o.
Use the following accepted information to show why this is always true.
Given: The sum of all interior angles in a triangle is always 180^o.



Try to create another diagram with explanation which shows why the five angles in a star will always add up to 180^o. (We are getting way ahead of ourselves with this one.)



Functions

Finding the nth term:

For many patterns and sequences, it is easy to find the next term. Finding the 105th term may be much more difficult. Most patterns can be defined by an equation.

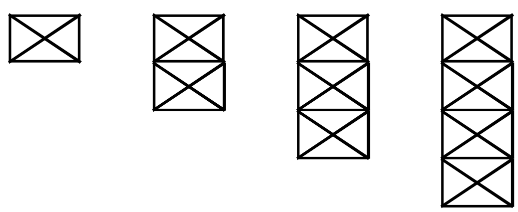
Examples: Write a function equation based on the following data:

term	1	2	3	4	5
value	-3	0	3	6	9

n	4	7	10	13	16
f(n)	-2	4	10	16	22

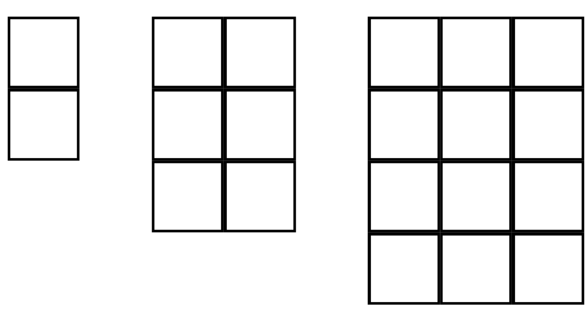
Patterns in geometry can be described similary:

Try to write an equation which describes this pattern to determine how many ‘toothpicks’ it takes to create each figure in the pattern.



It may help to create a table of values.

Some patterns are non-linear equations, but are still simple to write an equation for how many non-overlapping squares there will be in the nth figure.



You will need to recognize ways of counting and patterns to help you define more difficult patterns. In the figures above, how many straight ‘toothpicks’ are in each figure?

Write an equation that could help you discover how many lines would be needed to draw the 18th figure.

Quiz Review

Geometry 2.3

Find the next two terms:

100. **... 3, 8, 15, 24, 35, ___, ___, ...**

200. **... 5, 11, 16, 27, 43, 70, ___, ___ , ...**

300. **...** $\frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, \frac{7}{9}, \frac{4}{5}, \frac{9}{11},$ **___, ___, ...**

400. **... 360, 180, 120, 90, ___, ___, ...**

Inductive or Deductive?

100. Concluding that the sum of the angles in a regular hexagon is 720° using the known formula 180(n-2).

100. Sketching numerous quadrilaterals to show that it is impossible for a quadrilateral to have exactly three right angles.

100. Demonstrating that the difference of consecutive perfect squares will always be an odd number: $(n+1)^2 - (n)^2 =$

$(n^2 + 2n + 1) - n^2 =$

$2n + 1$ (an even number +1 is odd)

Use inductive reasoning to support each statement as true or false. If false, provide a counterexample.

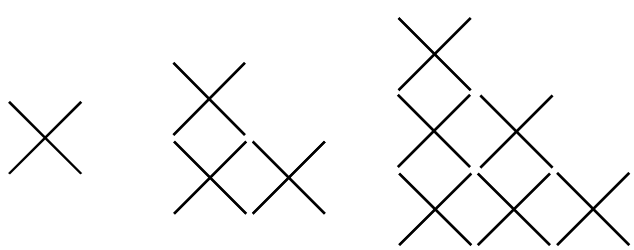
100. The number of diagonals in a regular polygon will always be greater than the number of sides.

200. A concave polygon must have at least four interior angles measuring less than 180°.

300. If you multiply consecutive prime numbers, double the result, and add one, the result will always be prime.

How many 'toothpicks' will be needed to make the n^th^ figure?

300.



Name_________________________ Period _____

Practice Quiz

Geometry 2.3

Find the next two terms in each sequence.
Show your reasoning below each problem.
All of these are standard mathematical sequences (not OTTFFSSENT…)

1. … 5, 7, 12, 19, 31, 50, ___, ___, …

1. _____ _____

2. … 4, 10, 18, 28, 40, 54, 70, ___, ___, …

2. _____ _____

3. … 7, 11, 13, 17, 19, 23, 29, ___, ___, …

3. _____ _____

4. … 80, 40, 120, 60, 180, 90, ___, ___, …

4. _____ _____

5. … $\frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$, ___, ___, …

5. _____ _____

Determine whether each conjecture was made by inductive or deductive reasoning:

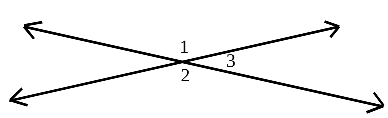
6. Gordon conjectured that adding any two odd numbers will always equal an even number by adding 3+5=8, 5+7=12, 17+9=26, all the way to numbers like 191+273=464.

6. Circle: inductive or deductive?

7. Vickie conjectured that the sum of all four angles in a square will always add up to 360^o, because all four angles are right angles therefore 4 times 90^o = 360^o.

7. Circle: inductive or deductive?

8. Evan conjectured that angles 1 and 2 below will always be equal because ∠1+∠3 = ∠2+∠3 (180^o) because they create a straight line, therefore subtracting ∠3 leaves ∠1 = ∠2.



8. Circle: inductive or deductive?

9. Amika conjectured that there are an infinite number of primes.
“If you take all the known prime numbers, multiply them together, and add one, then the number will either be prime, or it will have a factor greater than the largest known prime.”

9. Circle: inductive or deductive?

Name________________________ Period _____

Practice Quiz

Geometry 2.3

Use inductive reasoning to determine if you believe each statement is true or false. If you discover a statement that is false, provide a counterexample.

10. Multiplying two perfect squares will always yield another perfect square.

10. T/F ____ _______________

11. If x is odd, then x^2 must also be odd.

11. T/F ____ _______________

12. The sum of two integers will always be greater than either of the two integers.

12. T/F ____ _______________

13. The square root of any real number will always be less than the number itself.

13. T/F ____ _______________

14. It is possible to express any positive even number greater than two as the sum of two prime numbers.

14. T/F ____ _______________

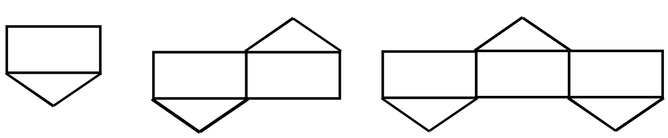
Write a function equation to find the n^{th} term in each sequence:

15.

n	1	2	3	4	5
f(n)	-9	-7	-5	-3	-1

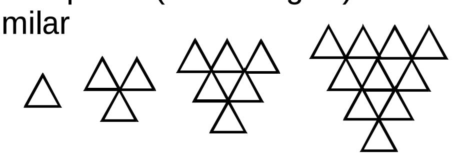
15. ____________________

16. How many 'toothpicks' will there be in the n^{th} figure?



16. ____________________

17. How many 'toothpicks' (not triangles) will there be in the n^{th} figure?
You have seen similar patterns before.



17. ____________________

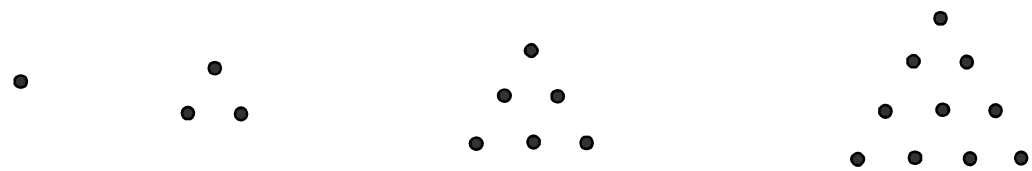
Pledge:

The 'Bowling Pin' Pattern

Geometry

Intro to geometric sequences:

n=1 n=2 n=3 n=4



f(n)=1 1+2 1+2+3 1+2+3+4...

$$f(n) = \frac{n(n+1)}{2}$$

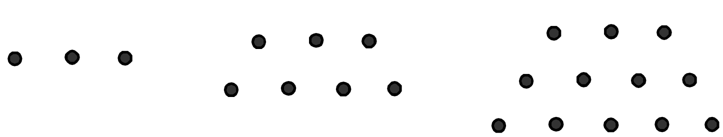
Today we will look at this function in a particularly useful way:
To find the number of pins in the nth figure, it is useful to look at the bowling pin pattern with numbers:

The nth figure will have **1 + 2 + 3 + 4 + 5 + ... + n pins.**
There are **n rows of pins.**
How could you find the **average number of pins in each row?**

To find the average number of pins in each row:
Add the first row (1) to the last row (n) and divide by 2.

Therefore, the number of pins is just the number of rows (n) times the average number of pins in each row (n+1)/2.
THIS ONLY WORKS IF THE PATTERN OF NUMBERS ADDED IS STRICTLY ARITHMETIC (5+7+9+11, but not 8+12+20+36+...)
Lets see how this can help write a different formula for the pattern below:

n=1 n=2 n=3



f(n)=3 3+4 3+4+5 3+4+5+...+(n+2)

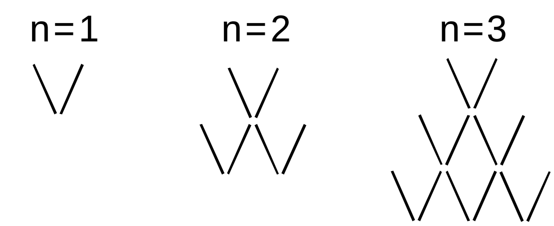
The ‘Bowling Pin’ Pattern

You should memorize the standard “bowling pin” function.

$$f(n) = \frac{n(n+1)}{2}$$

Instead of using the average method, it is also possible to simply MODIFY THE BOWLING PIN PATTERN.

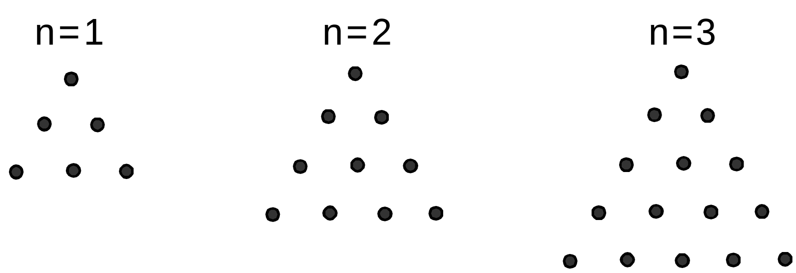
Example: How many toothpicks will there be in the nth figure?



You should recognize this as the bowling pin pattern, only doubled.

Therefore: $f(n) = \frac{2n(n+1)}{2} = n(n+1)$

It is also possible (though often confusing) to look at a pattern that is simple a shift of the standard pattern:



You should recognize that the pins in the first figure are the same as the pins in the 3rd figure of the standard bowling pin pattern. The pattern is simply shifted ‘up’ by 2. Modify the standard formula, replacing n with n+2:

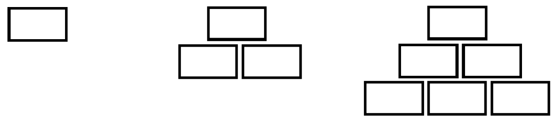
Therefore: $f(n) = \frac{n(n+1)}{2}$ becomes $f(n) = \frac{(n+2)(n+3)}{2}$

Functions

Practice: Write an equation for the number of given items in each pattern shown below.

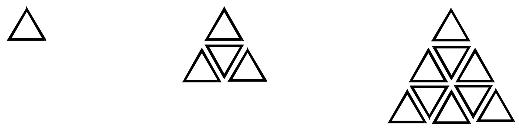
1. (bricks)

1st 2nd 3rd nth



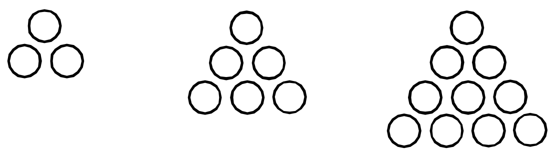
2. (triangles)

1st 2nd 3rd nth



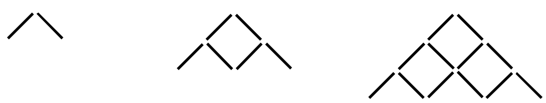
3. (circles)

1st 2nd 3rd nth



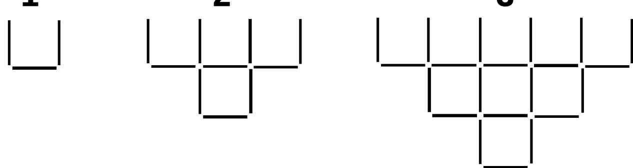
4. (sticks)

1st 2nd 3rd nth



5. (sticks)

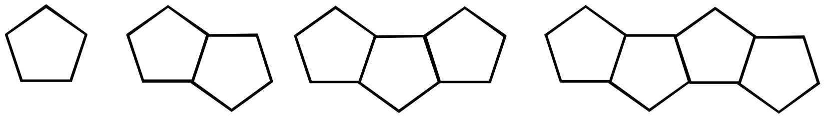
1st 2nd 3rd nth



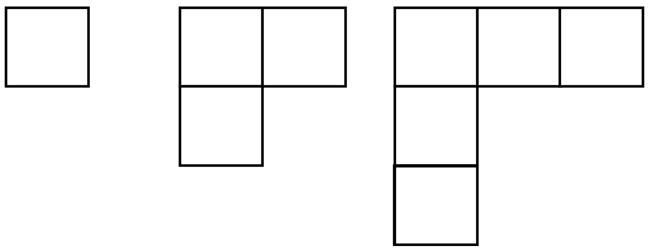
Functions

Practice: Write an equation to determine how many 'toothpicks' are required to create the *n* th figure in each pattern.

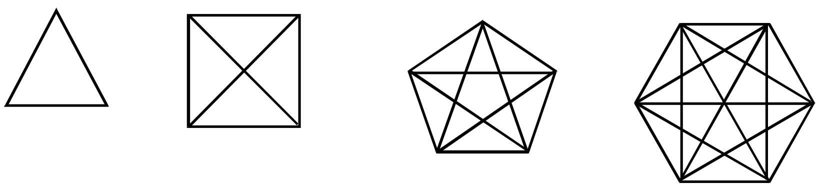
1.



2.



3.



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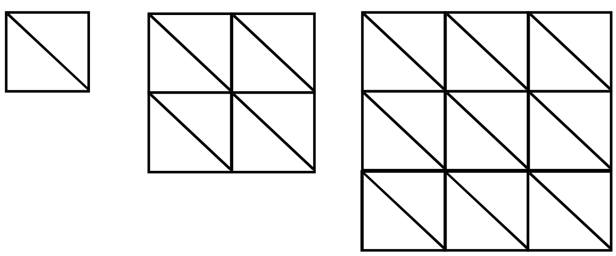
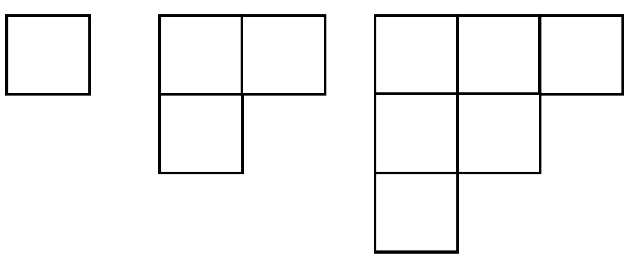


5.



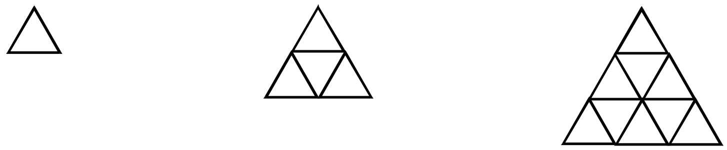
Name________________________ Period _____

Functions

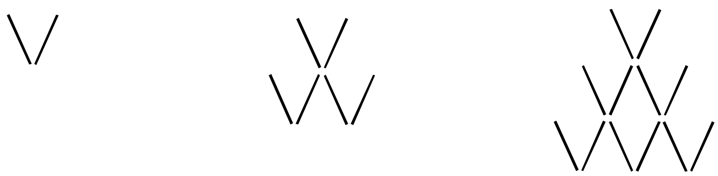
Geometry 2.3

Each pattern or sequence below can be defined by a mathematical function. Write a function to describe how many **toothpicks** are required to create the n^{th} figure in each:

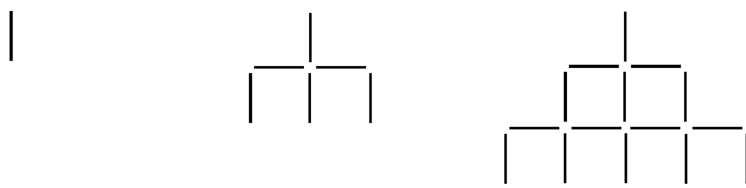
1^{st} 2^{nd} 3^{rd} n^{th}

1.  ________________

1^{st} 2^{nd} 3^{rd} n^{th}

2.  ________________

1^{st} 2^{nd} 3^{rd} n^{th}

3.  ________________

1^{st} 2^{nd} 3^{rd} n^{th}

4.  ________________

Solve by writing an equation:

5. How many chords are defined by n points on a circle?
(How many segments must be drawn to connect n points on a circle so that every point is connected to every other point)? ________________

6. Nine soccer teams play in a league. If every team plays every other team exactly once, how many total games must be played in the season?

____

7. Write a function that could be used to find the sum of n terms in the following sequence:
9 + 14 + 19 + 24 + 29 ... ________________

Name________________________ Period _____

Functions

Geometry 2.3

Once you have learned the 'bowling pin' pattern and how to recognize it, it should be easy to write a function for adding an arithmetic sequence. For each of the following, determine an equation that would help you find the nth term, then write a function that would allow you to add n terms in each sequence.

Ex. 1 + 2 + 3 + 4 + ... **nth term:** n

sum of the first n terms: $\frac{n(n+1)}{2}$

1. 5 + 2 + 3 + 4 + ... **n^{th} term:** ____________

sum of the first n terms: ______________

2. 15 + 30 + 45 + 60 + ... **n^{th} term:** ____________

sum of the first n terms: ______________

3. 4 + 5 + 6 + 7 ... **n^{th} term:** ____________

sum of the first n terms: ______________

4. 17 + 20 + 23 + 26 + ... **n^{th} term:** ____________

sum of the first n terms: ______________

5. -7 -11 -15 -19 ... **n^{th} term:** ____________

sum of the first n terms: ______________

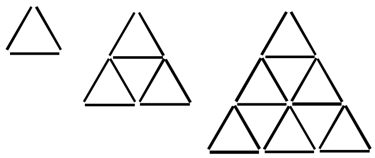
Name________________________ Period _____

Functions

Geometry 2.3

The following diagrams can be written as the sum of an arithmetic series (similar to 1+2+3...). Write the series, then write the function that will give you the number of toothpicks in the nth figure.

Ex.

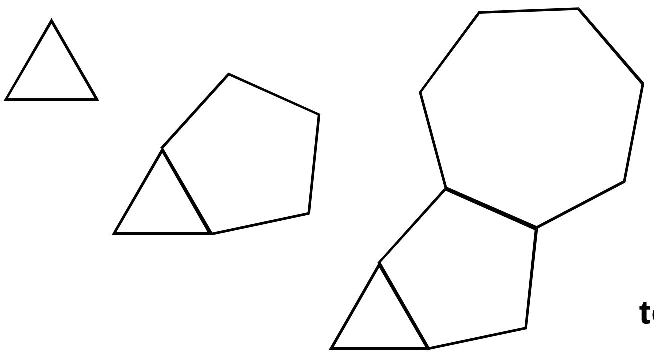


series: $3+6+9+12...$

toothpicks in the n^th^ figure: $\frac{3n(n+1)}{2}$

toothpicks in the 50^th^ figure: $75 \cdot 51 = 3,825$

6.

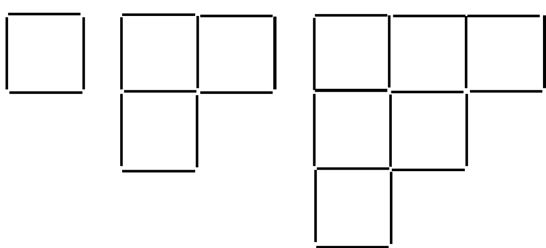


series: ____________

toothpicks in the n^th^ figure: ___________

toothpicks in the 50^th^ figure: ___________

7.

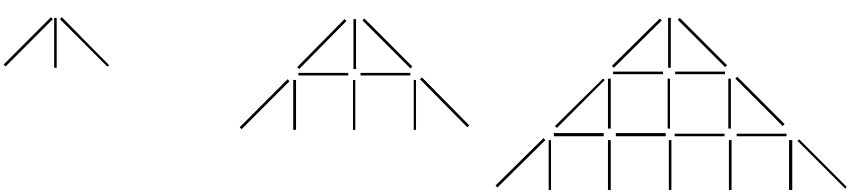


series: ____________

toothpicks in the n^th^ figure: ___________

toothpicks in the 50^th^ figure: ___________

8.



series: ____________

toothpicks in the n^th^ figure: ___________

toothpicks in the 50^th^ figure: ___________

Name_________________________ Period _____

Functions

Geometry 2.3

Practice: Write an expression that would represent the n^{th} term in each sequence below. Then, write a function equation that could be used to find the sum of the first n terms in each pattern.

Ex. **3 + 5 + 7 + 9 + ...**

n^{th} term: $2n + 1$

sum of the first n terms: $\frac{n[(2n+1)+3)]}{2} = n(n+2)$

1. **5 + 6 + 7 + 8 + ...**

n^{th} term: ____________

sum of the first n terms: ______________

2. **5 + 10 + 15 + 20 + ...**

n^{th} term: ____________

sum of the first n terms: ______________

3. **4 + 1 + 2 + 3 + 4 ...**

n^{th} term: ____________

sum of the first n terms: ______________

4. **6 + 8 + 10 + 12 + ...**

n^{th} term: ____________

sum of the first n terms: ______________

5. **-2 -4 -6 -8 ...**

n^{th} term: ____________

sum of the first n terms: ______________

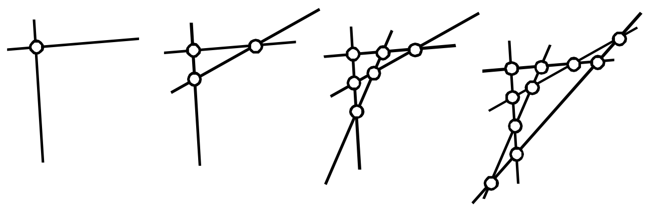
Name________________________ Period _____

Functions

Geometry

The following diagrams can be written as the sum of an arithmetic series (similar to 1+2+3...). Write the series, then write the function that will give you the number of dots/toothpicks in the nth figure.

6. (dots)

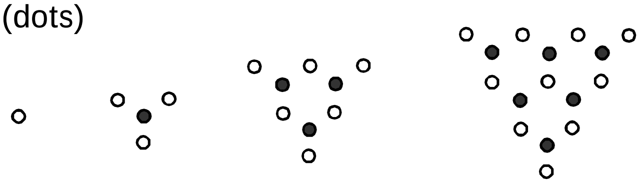


series: _____________

dots in the nth figure: ____________

dots in the 50th figure: ____________

7. (dots)

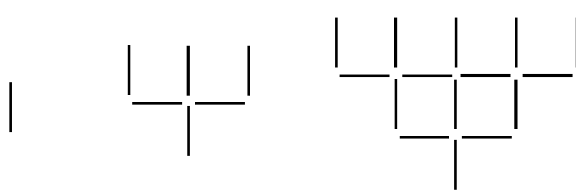


series: _____________

dots in the nth figure: ____________

dots in the 50th figure: ____________

8.

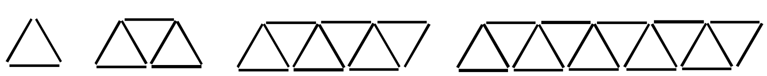


series: _____________

toothpicks in the nth figure: ____________

toothpicks in the 50th figure: ____________

9.



series: _____________

toothpicks in the nth figure: ____________

toothpicks in the 50th figure: ____________

Handshake Problems

Geometry

Consider the following...

Ten men meet for a bowling tournament, and each shakes the hand of every other man. How many handshakes occurred?

Six teams are in the same football conference. Each team must play every other team in the conference at home and away. How many games is this altogether?

Practice:

1. Seven lines are drawn on a sheet of paper so that each line crosses the other six, but no more than two lines intersect at each point. How many points of intersection are there?

2. Fifteen points are placed on the circumference of a circle. How many lines will it take to connect each point to every other point?

3. A number of couples attend a dinner party, and each person shakes the hand of everyone but his or her spouse. If there were 144 handshakes altogether, how many people attended the party?

Practice:

1. A bag contains jellybeans in 15 flavors. If two different flavors are combined, a new flavor is created. How many new flavors can be created by combining two different-flavored jelly beans?

2. In a chess league, how many games must be played so that each of the top 8 players plays each of the other players ranked in the top 8 three times?

3. Fifteen points are numbered on a circle 1 through 15. How many lines must be drawn to connect every pair of points whose sum is odd?

Method 1:

If you can recognize a pattern of numbers as a sum of numbers increasing linearly, sometimes it is easiest to simply

MODIFY THE BOWLING PIN PATTERN.

Example: 3+4+5+6+7...

This is the same as the bowling pin pattern, except that there are three pins missing from the beginning (1+2), and n is shifted 'up' two units.

Therefore: $\frac{n(n+1)}{2}$ **becomes** $\frac{(n+2)(n+3)}{2}-3$

Method 2:

The **NUMBER OF TERMS** times their **AVERAGE**.

Example (same): 3+4+5+6+7...

There are n terms, and the average of these terms is $\frac{\overset{\text{last}}{(n+2)} \overset{+}{+} \overset{\text{first}}{3}}{2}$

Therefore: $n \cdot \frac{(n+2)+3}{2} = \frac{n(n+5)}{2}$

Notice that our first equation $\frac{(n+2)(n+3)}{2}-3$ **equals** $\frac{n(n+5)}{2}$

Method 3:

Reasoning and application:

Example: For the world table tennis tournament team, the United States wants to send its best two-man team to the World Championships.

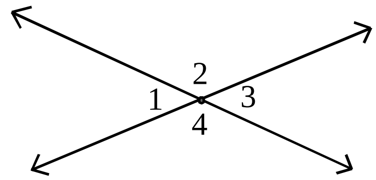
a. How many possible two-man teams can be created from the top 20 players in the country?

b. If ten teams are created and each team must play against every other team twice, how many games of table tennis must be played?

Challenge: If every possible two man team played every other possible two man team once, how many games must be played?

Angle Relationships

We have discussed the relationships between certain types of angles. Complete the sentences below with the diagram in your notes.



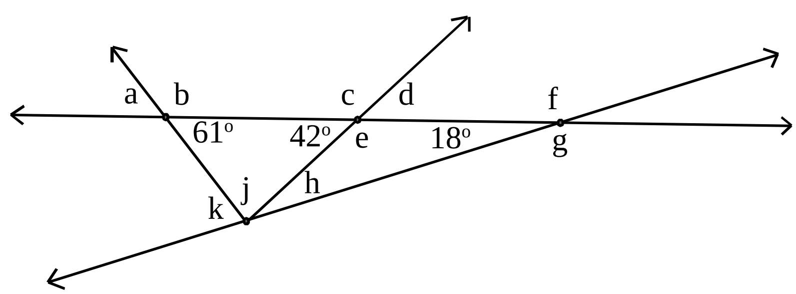
1. Angles 1 and 2 are __________ angles whose sum is _______ degrees.
2. Angles 1 and ___ are vertical angles whose measures are __________.

The **Linear Pair Conjecture** states that if two angles form a linear pair, then ______.
The **Vertical Angles Conjecture** states that if two angles are vertical, then _____.

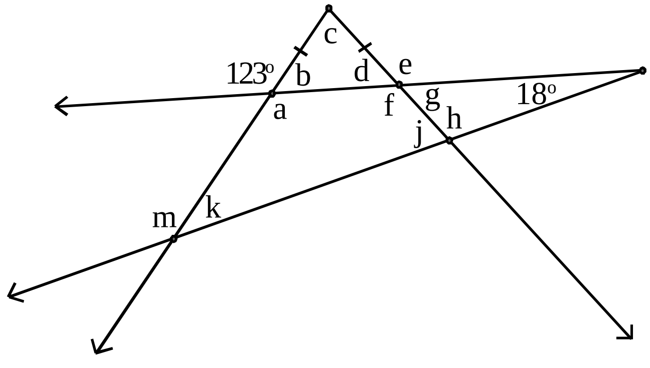
How could you demonstrate each conjecture inductively?
How could you demonstrate the vertical angles conjecture deductively, using the linear pair conjecture and some basic algebra? Do it with a paragraph proof.

Practice: Use what you know about linear and vertical angles, plus what you know about the sum of the interior angles in polygons to discover the missing measures below.

1.



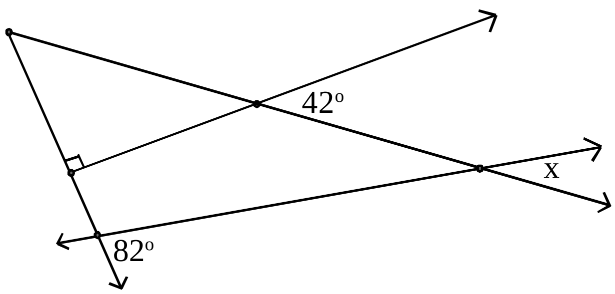
2.



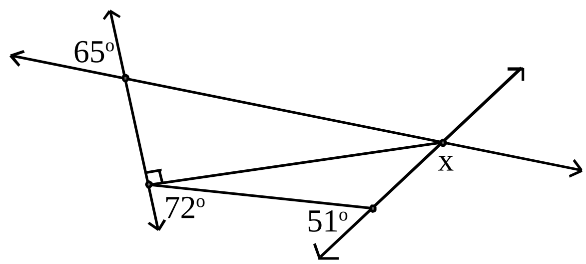
Angle Relationships

Find the missing angle in each diagram.

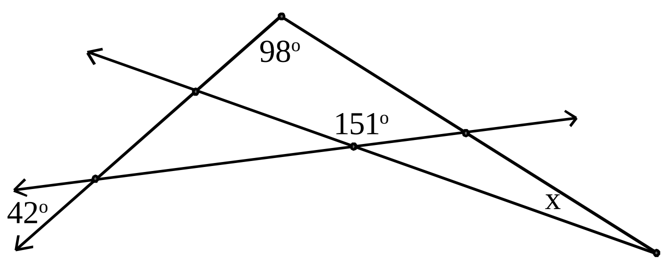
1.



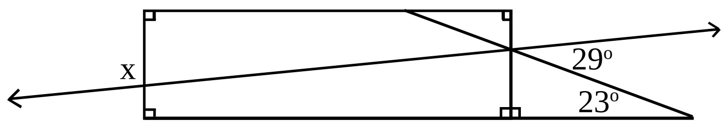
2.



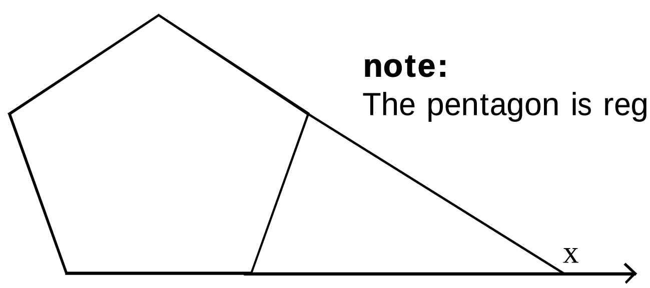
3.



4.



5.



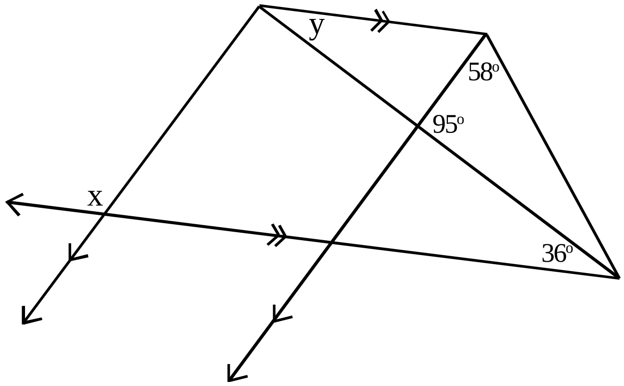
note:
The pentagon is regular.

Name__________________________ Period _____

Angle Relationships

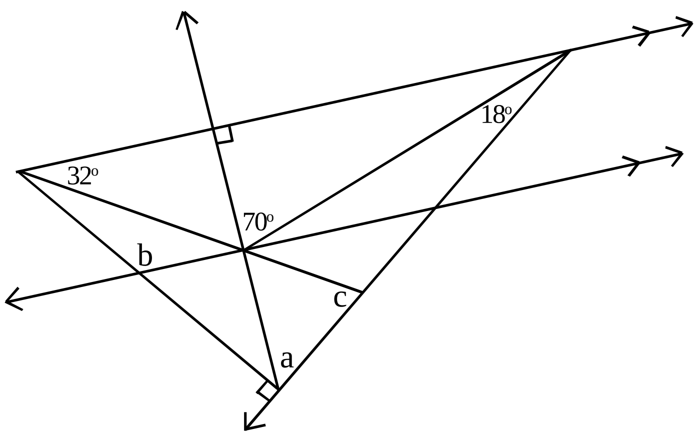
Geometry 2.6

Find the missing angles in each diagram.



x ______

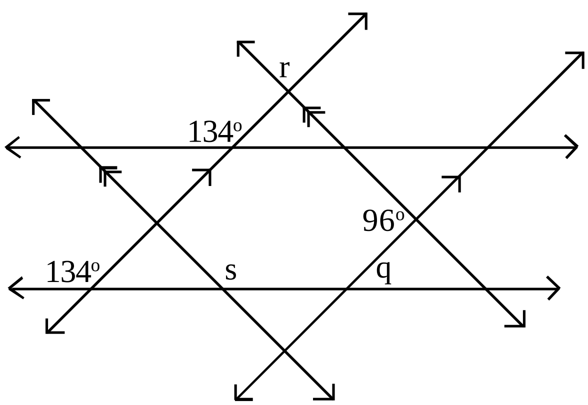
y ______



a ______

b ______

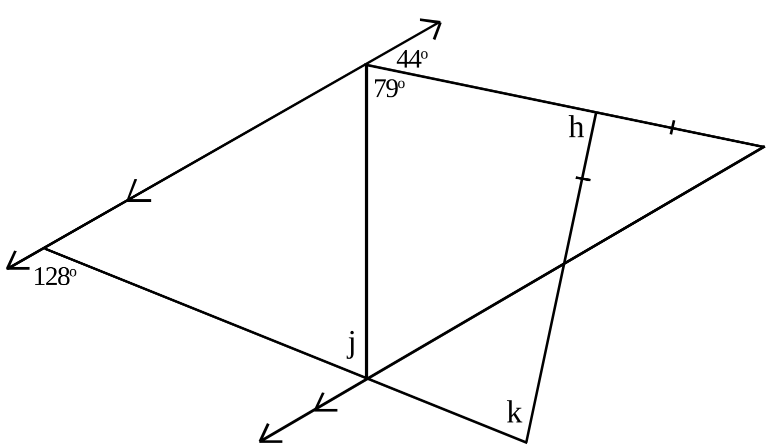
c ______



q ______

r ______

s ______



h ______

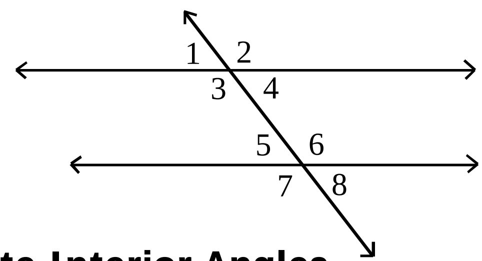
j ______

k ______

Parallel Lines

Geometry 2.6

Parallel lines are in the same plane but never intersect.
When a line intersects a pair of parallel lines, it is called a **transversal**.
Three types of angles are created:

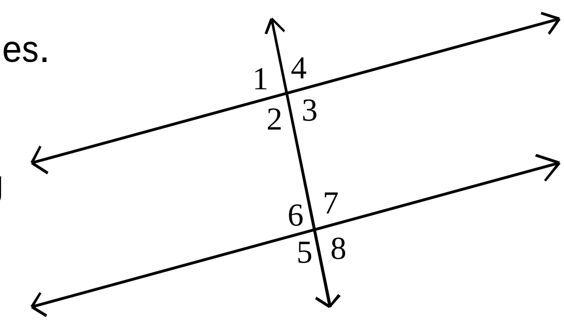


Angles 1 and 5 are **Corresponding Angles.**
Name three other pairs of corresponding angles.

Angles 5 and 4 are **Alternate Interior Angles**.
Interior because they are 'inside' the parallel lines, alternate because they are on opposite sides of the transversal.
Name the other pair of alternate interior angles.

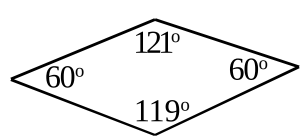
Angles 1 and 8 are **Alternate Exterior Angles**.
Exterior because they are outside the parallel lines, alternate because they are on opposite sides of the transversal.
Name the other pair of alternate exterior angles.

All three types above are congruent angles. 3 conjectures:
Corresponding angles conjecture (CA conjecture).
Alternate interior angles conjecture (AIA conjecture).
Alternate Exterior angles conjecture (AEA conjecture).



1. Name four congruent angles.
2. Name the other four congruent angles.
3. Name four angles which are supplementary to angle 1.
4. Angles 3 and ___ are corresponding angles.
5. Angles 3 and 6 are ___ ___ angles.
6. Angles 1 and 8 are ___ ___ angles.
7. Challenge: How many pairs of supplementary angles are there?

Converse: If corresponding angles (or AIA or AEA) are congruent, then the lines are parallel.
Are any of the lines in this figure parallel? How can you tell?



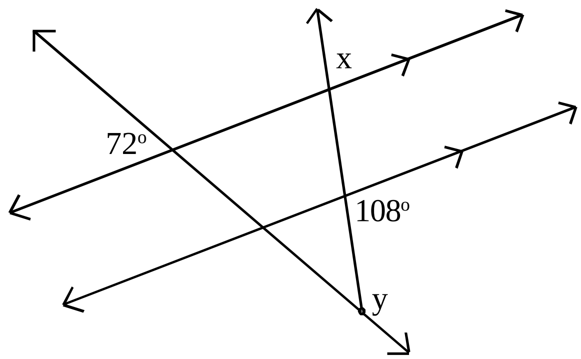
Name________________________ Period _____

Angle Relationships

Geometry 2.6

Find the missing angle in each diagram.

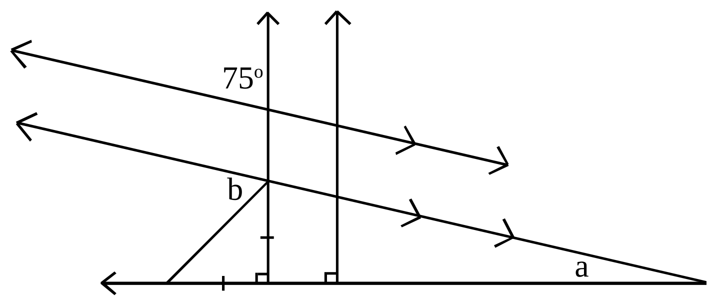
1-2.



x ______

y ______

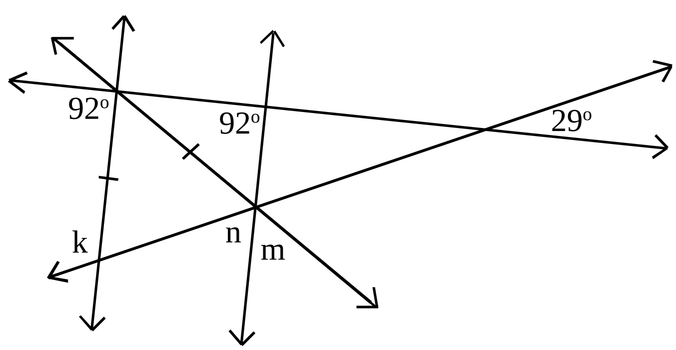
3-4.



a ______

b ______

5-7.



k ______

m ______

n ______

Name________________________ Period _____

Pattern Relationships

Geometry 2.6

Solve each, and create a formula that would help solve for any number given.

8. Dhruv is playing in a badminton league that has a total of 24 competitors.
How many matches will the league need to schedule if every player needs to face every other person in the league?

8a. __________

If a 25th competitor were added to the schedule, how many MORE games would need to be played?

8b. __________

9. Vanessa needs to add the numbers in the following arithmetic sequence: 7+9+11…+100+103+105.

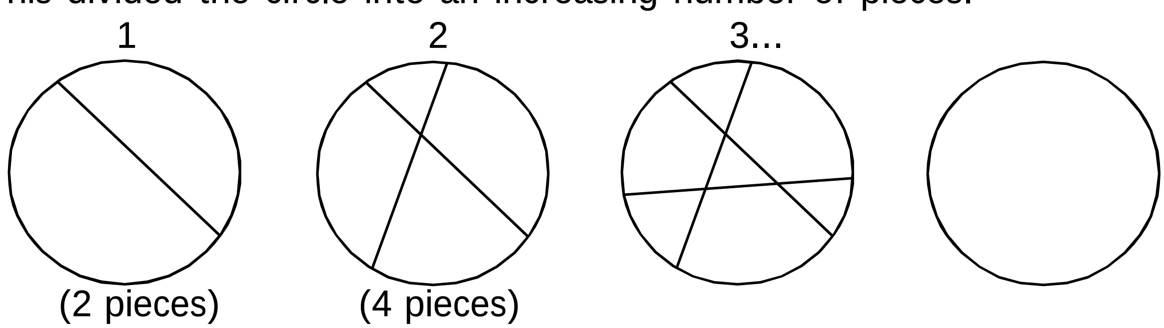
How many numbers are in the sequence?

9a. __________

What is the sum of all the numbers?

9b. __________

10. Andrew drew chords in a circle so that each new chord crossed each previously drawn chord, but no three chords intersect at the same point. This divided the circle into an increasing number of pieces.



How many pieces is the circle divided into by 5 chords?

10a. __________

How many pieces is the circle divided into after the nth chord is drawn? (Write the equation.)

10b. f(n)=__________________

Chapter 2 Review

Find the next terms in each sequence below:

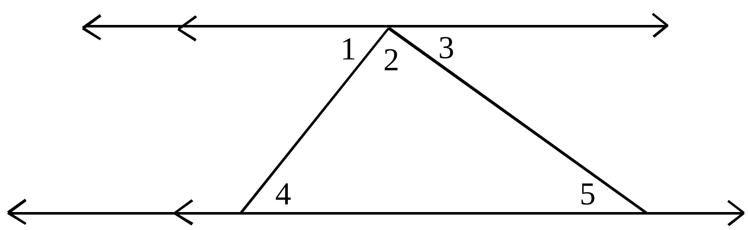
1. ... 8, 4, 12, 6, 18, 9, ___, ___, ...

2. ... -3, 4, 1, 5, 6, 11, ___, ___, ...

3. ... 0.5, 2, 4.5, 4, 12.5, 6, 24.5, ___, ___, ...

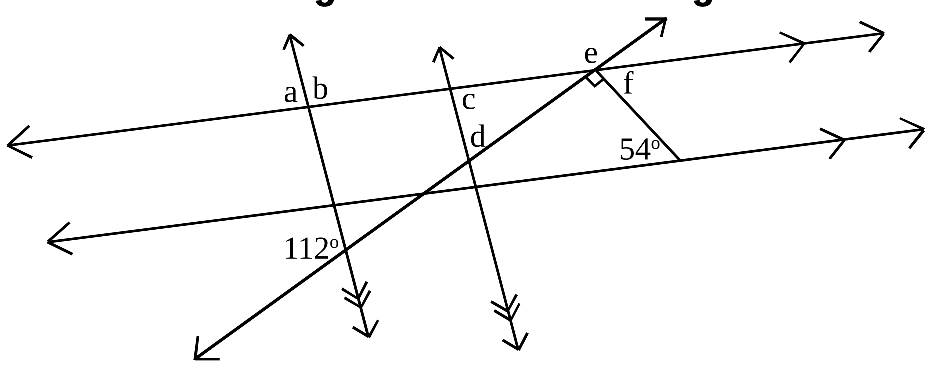
4. ... -3, -5, -7, -11, -13, -17, ___, ___, ...

Describe a **deductive method** of proving that the measure of the interior angles in a triangle must equal 180 degrees using the diagram below:

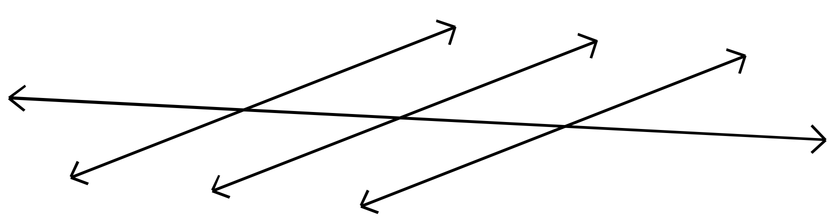


Use a paragraph proof and/or algebra, and include what we have learned this week in your proof.

Find each of the angles labeled in the figure below:



How many pairs of congruent angles are there in the figure below? If a transversal crosses through n lines, how many pairs of congruent angles will be created?

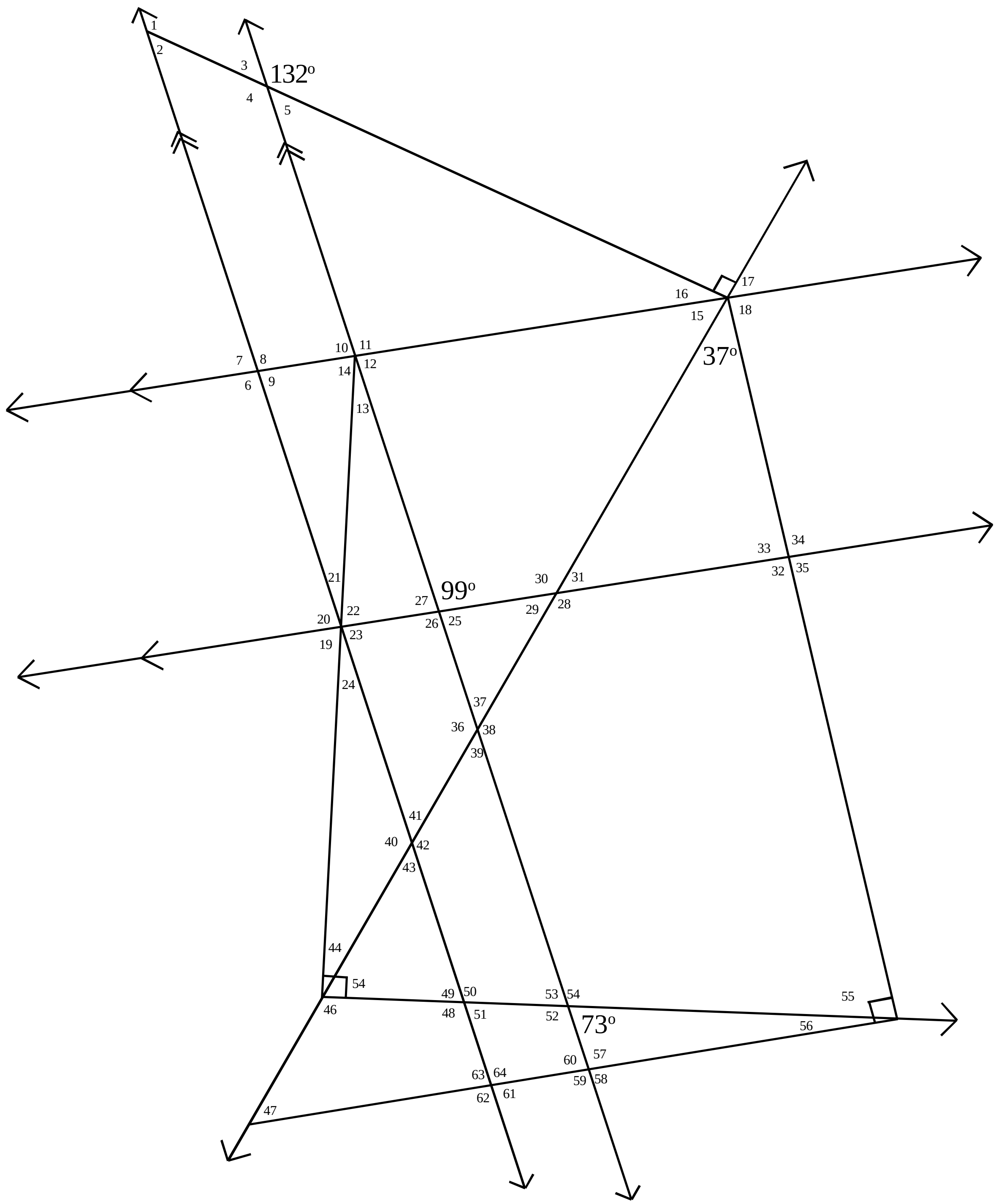


Name________________________ Period _____

Angles

Geometry 2.6

Find each missing angle in the figure below:



Fil in some of the tougher ones below:

#14 _____ #17 _____ #21 _____ #31 _____ #56 _____

Name________________________ Period _____

Practice Test

Geometry 2.6

Find the next two terms in each sequence. (not all are arithmetic sequences)

1. ... 1, 4, 9, 61, 52, 63, 94, 46, ___, ___, ...

1. _____ _____

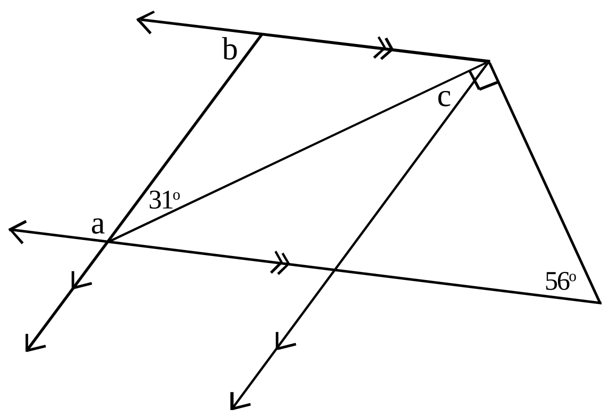
2. ... 15, 13, 10, 6, 1, -5, -12, ___, ___, ...

2. _____ _____

3. ... ___, ___, 2, -3, -1, -4, -5, -9, -14, ...

3. _____ _____

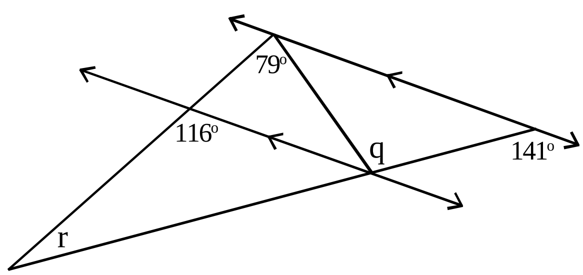
Find each missing angle measure in the figures below. (angles not to scale)



4. a _____

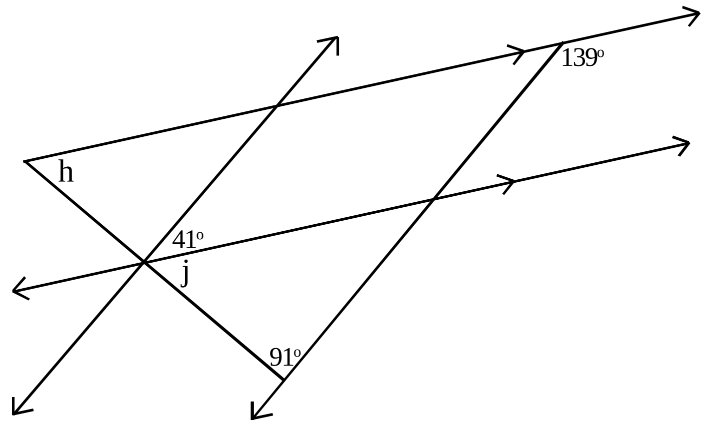
5. b _____

6. c _____



7. q _____

8. r _____



9. h _____

10. j _____

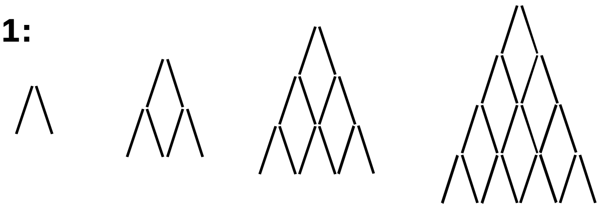
Name_________________________ Period _____

Practice Test

Geometry 2.6

Answer the questions that follow for each pattern below:

Pattern 1:



11. How many toothpicks will be in the nth figure?

11. f(n)=____________

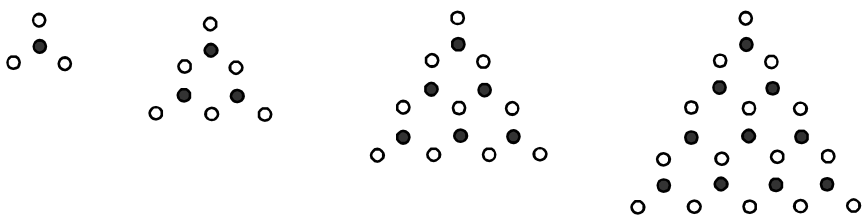
12. How many toothpicks will be in the 20th figure?

12. ____________

13. How many *diamonds* (small only) will be in the nth figure?

13. f(n)=____________

Pattern 2:



14. In the nth figure, how many white dots will be on the bottom row?

14. ____________

15. How many white dots will there be in the nth figure?

15. f(n)=____________

16. How many white dots will there be in the 25th figure?

16. ____________

17. How many black AND white dots will there be in the nth figure?

17. f(n)=____________

18. How many black AND white dots will there be in the 50th figure?

18. ____________

Answer:

:

19. How many handshakes will occur between 15 strangers if each shakes every other person's hand?

19. ____________

20. How many *more* handshakes will occur if three new strangers enter?

20. ____________