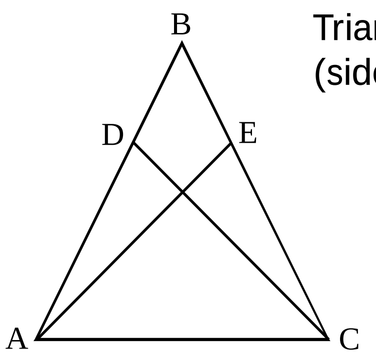
Triangle Basics

Geometry 4.0

First: Some basics you should already know.

1. What is the sum of the measures of the angles in a triangle?
   Write the proof (Hint: it involves creating a parallel line.)

2. In an isosceles triangle, the base angles will always be ________.
   The proof of this generally involves some information we will review today, but here it is two ways:

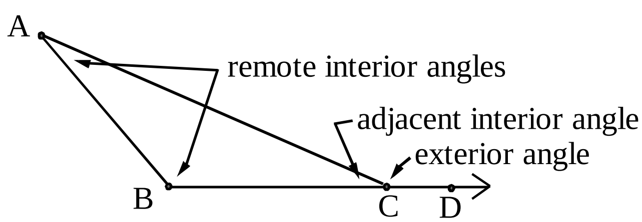


Triangle ABC is congruent to Triangle CBA (side-angle-side) therefore angle A = angle C.

Not satisfied? Add some lines:
AD = CE
Triangle ABE = Triangle CBD (SAS)
Therefore triangle CAD = ACE (subtraction)
Which makes angle BAC = BCA

3. If exactly two angles in a triangle are equal then it must be _________.
   (this is the converse of #2)

4. What is the relationship between an exterior angle of a triangle and the sum of the remote interior angles? Prove with just a sentence or two.



5. In triangle XYZ: XY=6inches, YZ=9 nches, and XZ=11 inches.
   Which is the largest angle: X, Y, or Z? The smallest?

6. Which of the following sets of numbers could NOT represent the three sides of a triangle?

3-4-5 5-12-13 8-15-20 16-17-40 10-10-17

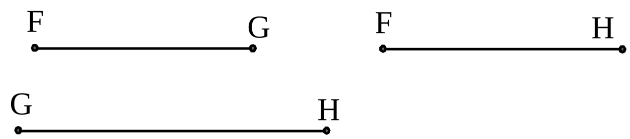
7. How many scalene triangles have sides of integral (integer) length and perimeter less than 15?

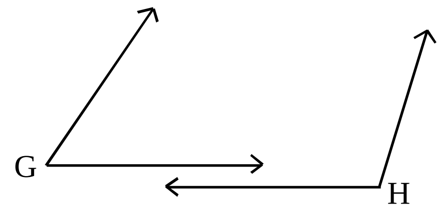
Name________________________ Period _____

Triangle Congruencies

Geometry 4.4

You have probably already heard of most of the triangle congruence short-cuts. Today we will construct several triangles to demonstrate the shortcuts we can use to show two triangles are congruent.





Figures are considered **congruent** if they are exactly the same. If you can slide, rotate, or reflect one figure so that it is exactly the same as another, the two figures are considered congruent.

1. ___**SSS:** Side-Side-Side
Use the three sides above to construct a triangle (begin with FH). Compare it to the ones your classmates drew. Does SSS demonstrate congruence?

2. ___**SAS:** Side-Angle-Side
Use FG, angle G and GH to construct a triangle. Compare it to the triangle your classmates drew. Does SAS demonstrate congruence?

3. ___**ASA:** Angle-Side-Angle
Use angle G, segment GH, and angle H to construct a triangle. Compare it to the triangle your classmates drew. (Is **AAS** a congruence shortcut? Why or why not?)

3+. ___**AAS:** Side-Angle-Angle

4. ___**SSA:** Side-Side-Angle
Use angle G, segment GH, and segment FH to construct a triangle. Compare it to the triangle your classmates drew. Does SSA demonstrate congruence? Is it possible to draw more than one triangle using angle G, segment GH, and segment FH?

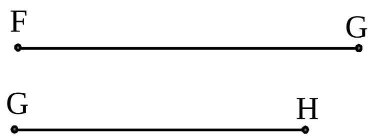
Name________________________ Period _____

Triangle Congruencies

Geometry 4.5

HL and LL congruence:

Use the following segments again.



1. ___**LL:** Leg-Leg (For right triangles.)

Construct Right angle FGH. Connect FH. Compare your triangle to the ones your classmates drew. Which congruence shortcut is this identical to?

2. ___**HL:** Hypotenuse-Leg (For right triangles.)

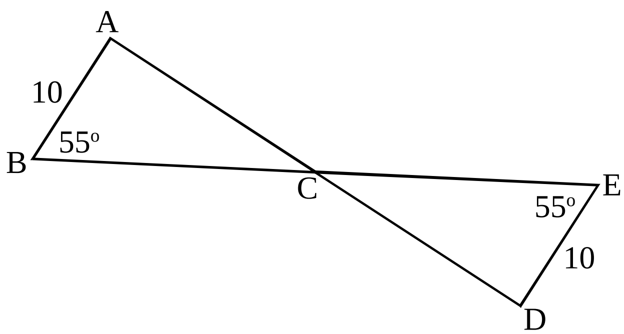
Construct right angle H on segment GH. Use length FG to complete right triangle FGH. Compare your triangle to the ones your classmates drew. Is this similar to any of the congruence shortcuts on the opposite side of this page?

Using Congruence Shortcuts

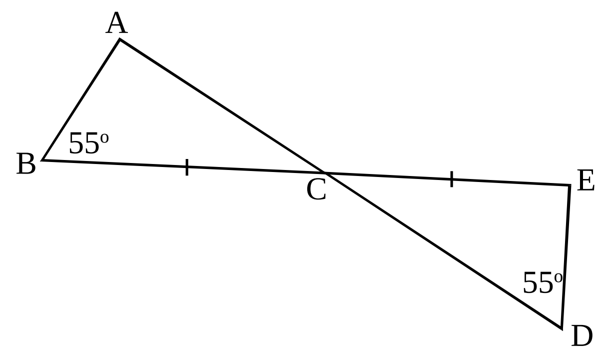
Geometry 4.5

Determine which of the following pairs of triangles are congruent and why: Triangles are not necessarily to scale.

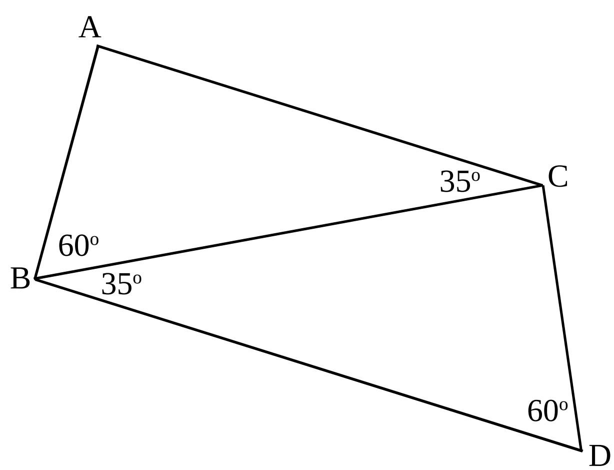
$\triangle ABC \cong \triangle$___ by ___.



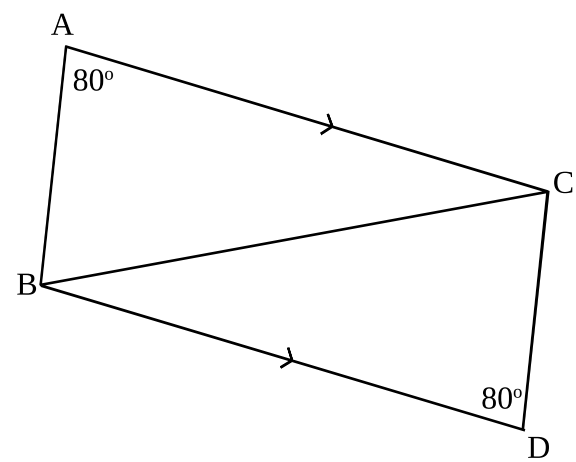
$\triangle BAC \cong \triangle DEC$?



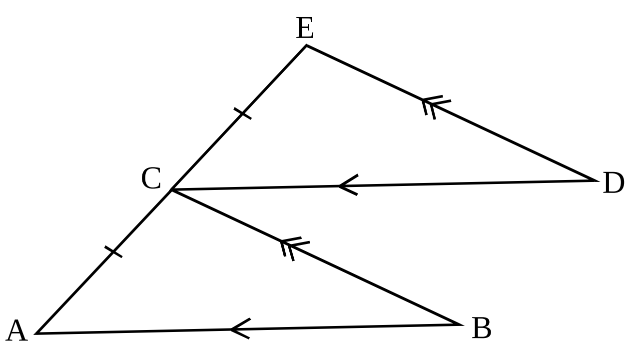
$\triangle BCD \cong \triangle BAC$?



$\triangle BAC \cong \triangle CDB$?

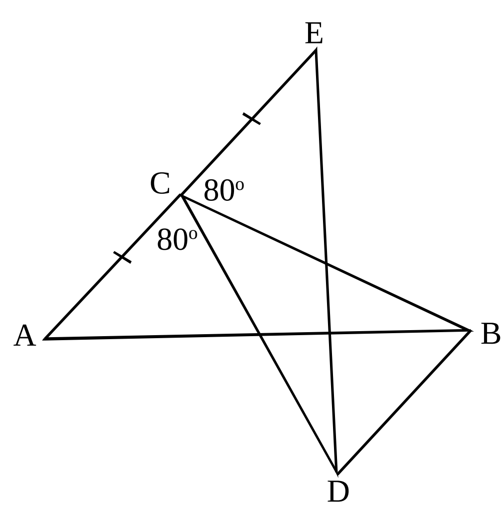


$\triangle ABC \cong \triangle CDE$?



$\angle CDB \cong \angle CBD$

$\triangle ACB \cong \triangle ECD$?

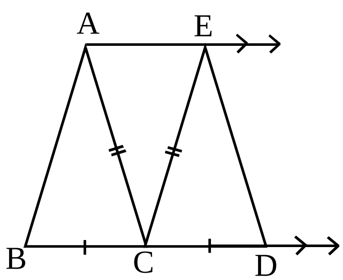


Using Congruence Shortcuts

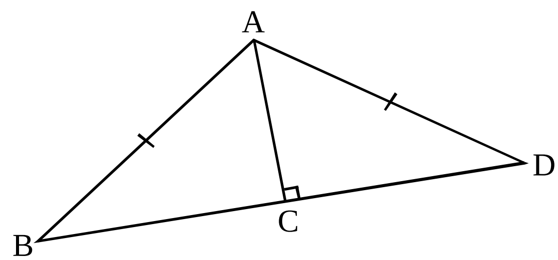
Geometry 4.5

Determine which of the following pairs of triangles are congruent and why: Triangles are **not** necessarily to scale.

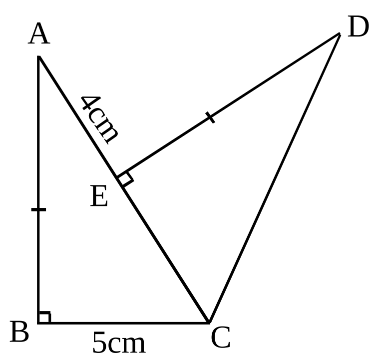
$\triangle ABC \cong \triangle$___ by ___.



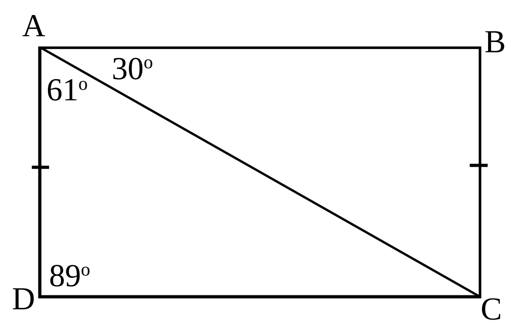
$\triangle ABC \cong \triangle ADC$?



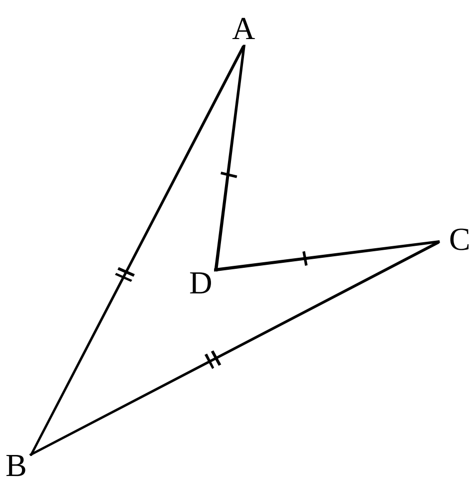
$\triangle CBA \cong \triangle CED$?



$\triangle ADC \cong \triangle CBA$?



$\angle A \cong \angle C$?

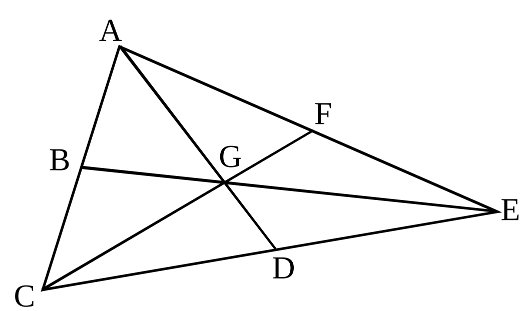


G is the centroid.

AD=CF

$\triangle AGF \cong \triangle CGD$?

$\triangle ADE \cong \triangle CFE$?



Name________________________ Period _____

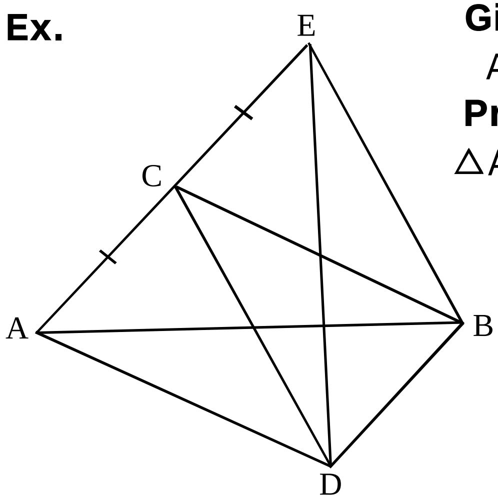
Proof/ Beginning Proof Writing

Geometry 4.5

Mathematical Proof takes an accepted set of facts and properties to demonstrate something to be true.

In a **two-column proof,** statements are made on the left and justifications are made on the right.

Ex.



Given:
$AE \parallel DB$ and $\angle ECB \cong \angle ACD$
Prove:
$\triangle ACD \cong \triangle ECB$

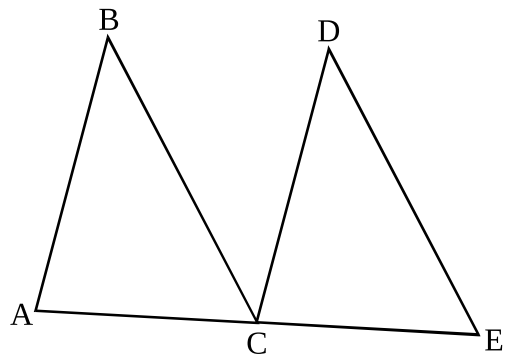
1. $\overline{AC} \cong \overline{EC}$	1. Given
2. $\angle ECB \cong \angle ACD$	2. Given
3. $AE \parallel DB$	3. Given
4. $\angle ACD \cong \angle CDB$	4. AIA
5. $\angle ECB \cong \angle CBD$	5. AIA
6. $\angle CDB \cong \angle CBD$	6. Transitive Property of Congruence
7. $\angle CDB$ is isosceles.	7. Base Angles are Congruent (Converse of Isos. $\triangle$Theorem)
8. $\overline{CD} \cong \overline{CB}$	8. Definition of Isosceles $\triangle$
9. $\triangle ACD \cong \triangle ECB$	9. SAS (1,2,8)

Some common justifications you will be using in your proofs:
Alternate Interior Angles (AIA)
Corresponding Angles (CA)
Definition of ______. (midpoint, square, kite, vertical angles, bisector, etc.)
SSS, ASA, SAS, SAA, HL, LL
Same Segment or **Same Angle** (ex. If you said $\overline{BD} \cong \overline{DB}$. This will later be called the Reflexive Property of Congruence, but that is not necessary now.)
Vertical Angles
Linear Pair
etc.
On the back, record any new justifications that we learn so that you can have a list to use when writing proofs.

CPCTC

If you can show that two triangles are congruent, then their corresponding parts are also congruent.

CPCTC: Corresponding Parts of Congruent Triangles are Congruent
We will use this shortcut when writing Two-Column Proofs.



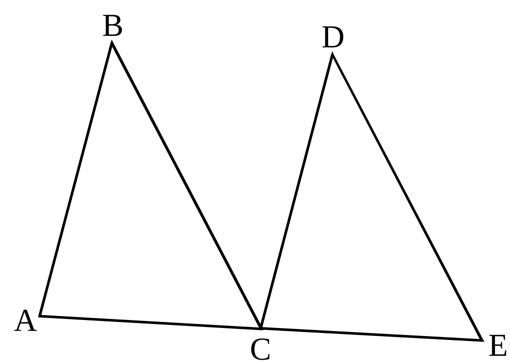
Given: C is the midpoint of segment AE.
AB and CD are parallel.
Angle B and Angle D are congruent.
Prove: BC=DE.

In a two column proof, statements are made in the left column, and justifications for those statements are on the right.

1. Begin with the given information.
2. Work through the diagram to determine whether the conclusion can be reached.
3. Organize the steps carefully as in the example below, including the given information.

EX.

Given: C is the midpoint of segment AE .
AB and CD are parallel.
Angle B and Angle D are congruent.
Prove: BC=DE.



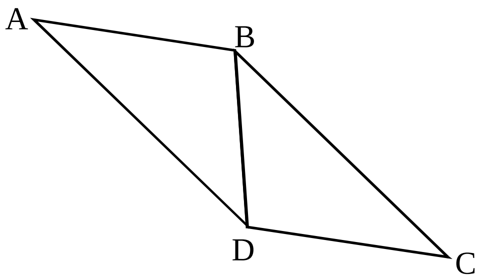
1. C is the midpoint of AE.	1. Given
2. AC=CE	2. Definition of midpoint
3. AB and CD are parallel.	3. Given
4. $\angle$ECD =$\angle$CAB	4. Corresponding angles
5. $\angle$B =$\angle$D	5. Given
6. $\triangle$ABC $\cong$ $\triangle$CDE	6. SAA congruence (# 2, 4, 5)
7. BC = DE	7. CPCTC

CPCTC

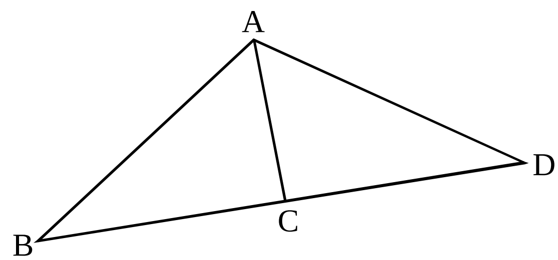
Geometry 4.6

Write a two-column proof for each:

Given:
$AB \parallel DC$
$\angle DBC \cong \angle BDA$
Prove: $\angle A = \angle C$

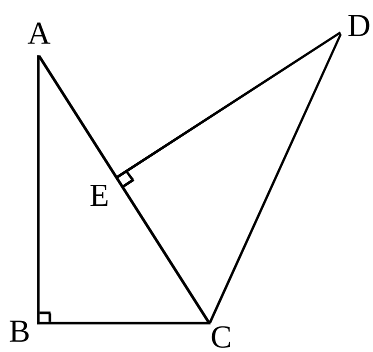


Given:
$AB = AD$
AC bisects $\angle BAD$
Prove: $\angle ACB = 90^{o}$

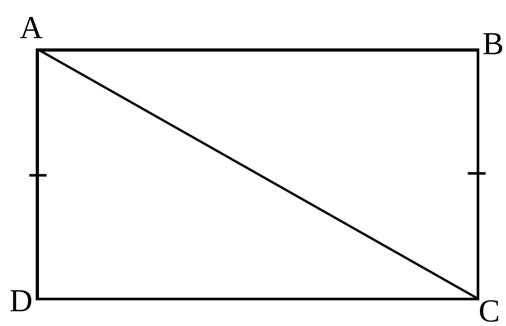


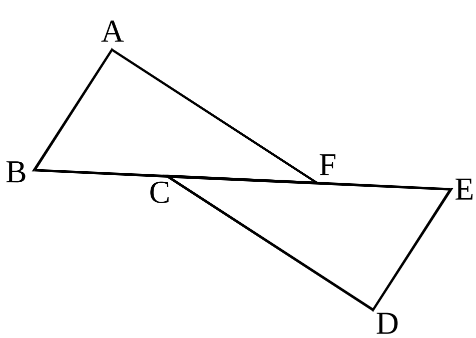
For each problem below, some of the given information is included in the diagram.

Given:
$AB = ED$
CE bisects $\angle BCD$
Prove: $BC = EC$



Given:
$AD \parallel BC$
Prove: $AB = CD$





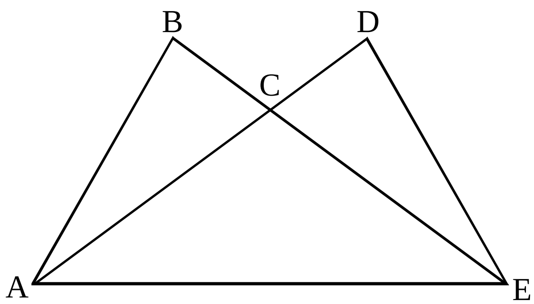
Given:
C is the midpoint of BF
F is the midpoint of CE
$AB \parallel DE$
$\angle A = \angle D$
Prove: $AF = CD$

Flowchart Proofs

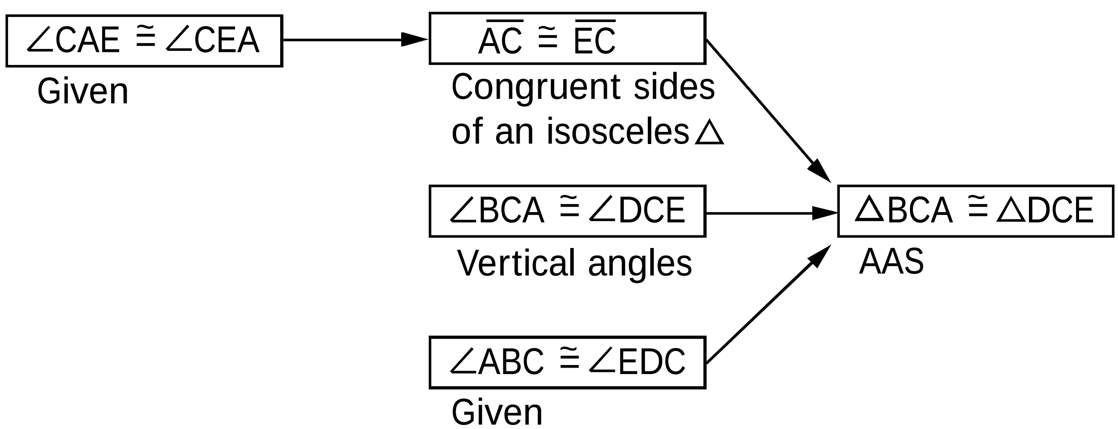
Flowcharts can be used to explain the logical organization of a proof.

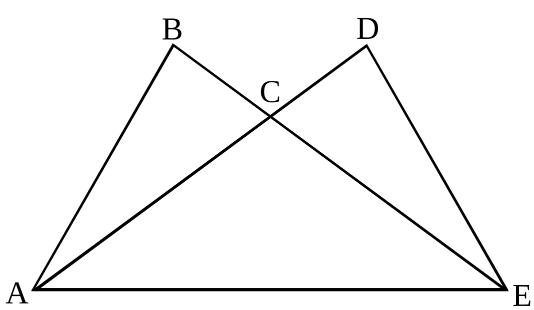
In a flowchart proof, statements are placed within boxes, with the justification below the box.

Arrows connect statements.
The arrow can be read as the word “therefore.”



Given:
$\angle ABC \cong \angle EDC$
$\angle CAE \cong \angle CEA$
Prove: $\triangle BCA \cong \triangle DCE$





Given:
$\triangle BCA \cong \triangle DCE$
Prove: $\triangle ADE \cong \triangle EBA$

This can be done many ways, try to find the easiest.

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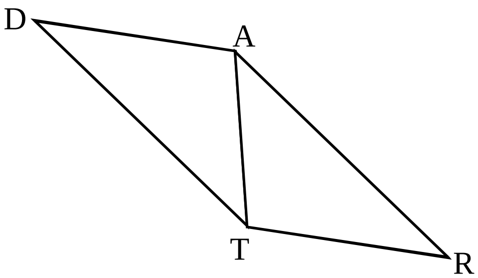
Practice Proofs Half-Quiz

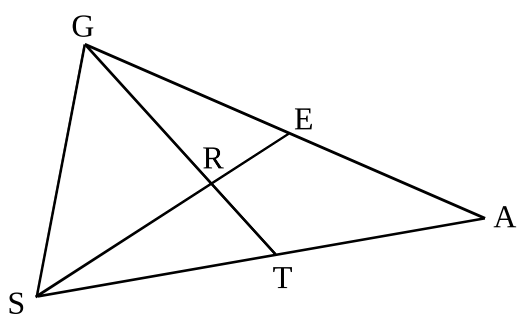
Geometry 4.7

Complete the following Proofs by filling in the missing blanks:



Given:
$DT \parallel AR$
$\angle D \cong \angle R$
Prove: $\overline{DA} \cong \overline{TR}$

	Statements		Reasons
1.	$DT \parallel AR$	**1.**	______________
2.	______________	**2.**	Alternate Interior Angles
3.	______________	**3.**	______________
4.	$\overline{TA} \cong \overline{AT}$	**4.**	______________
5.	______________	**5.**	AAS Congruence (3, 2, 4)
6.	______________	**6.**	CPCTC



Given:
$\triangle GES \cong \triangle STG$
$\angle ESA \cong \angle TGA$
Prove: $\overline{SA} \cong \overline{GA}$

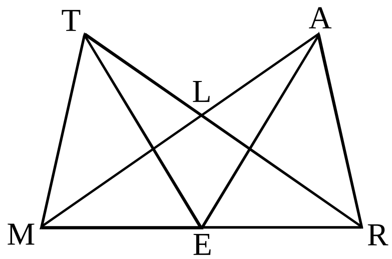
	Statements		Reasons
1.	$\triangle GES \cong \triangle STG$	**1.**	______________
2.	______________	**2.**	CPCTC
3.	______________	**3.**	Given
4.	______________	**4.**	Same Angle
5.	$\triangle ASE \cong \triangle AGT$	**5.**	______________
6.	______________	**6.**	CPCTC

Name________________________ Period _____

Practice Proofs Half-Quiz

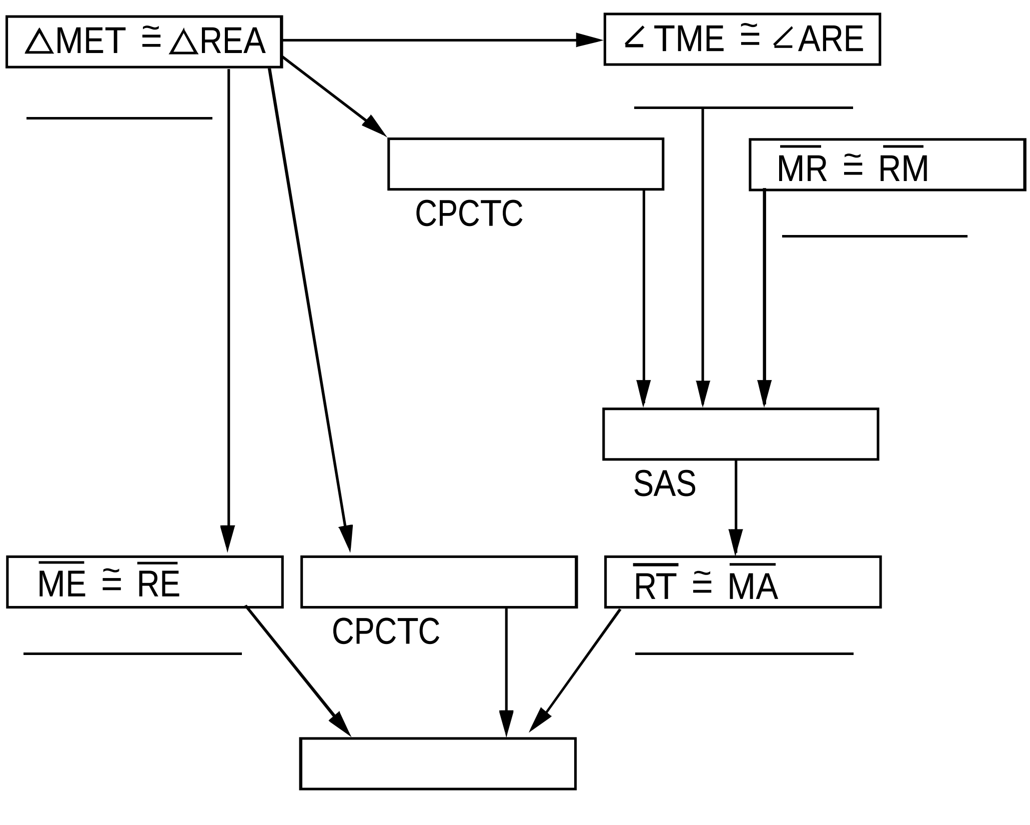
Geometry 4.7

Complete the following Proof by filling in the missing blanks:



Given:
$\triangle MET \cong \triangle REA$

Prove: $\triangle MEA \cong \triangle RET$



Special Proofs

One shortcut:
For several the following proofs, we will shorten some steps by using the following theorem:
If two angles are both linear and congruent, then they are right angles.
In our proofs, the justification will look like:
1. $\angle XYZ = 90^o$ 1. Congruent Linear Angle (with $\angle WYZ$).

Proofs involving special triangles.
Use a two-column or flowchart proof for each:

1. Prove that the bisector of the vertex angle in an isosceles triangle is also the median.

2. Prove that the altitude from the vertex of an isosceles triangles is also an angle bisector.

3. In a given circle, prove that if a radius bisects a chord then the chord and radius are perpendicular.

4. Explain (too long for a formal proof) why the incenter, circumcenter, orthocenter, and centroid are all the same point in an equilateral triangle.

Proofs involving quadrilaterals.
Use a two-column or flowchart proof for each:

1. Prove that the diagonals in a square are angle bisectors.

2. Prove that the diagonals in a parallelogram are of equal length.

3. Explain how you could prove that the diagonals in a parallelogram bisect each other.

4. Prove that the diagonals in a rhombus are perpendicular (to each other).

Confused about one of these?

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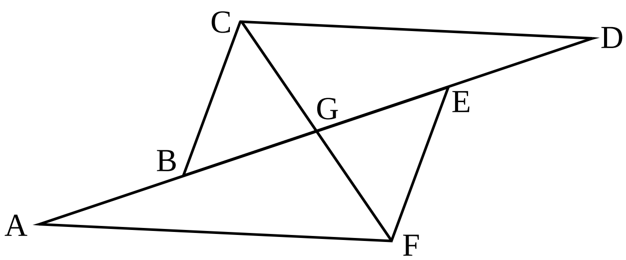
CPCTC and Proofs

Geometry Re

Prove each of the following using CPCTC.

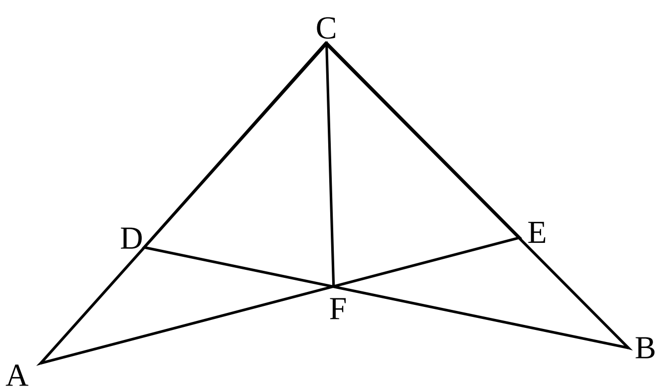
Write a two-column proof for:



Given: $\triangle BCD \cong \triangle EFA$

Prove: $\overline{CG} \cong \overline{GF}$

Complete a flowchart proof for the following:



Given: $\triangle DCF \cong \triangle ECF$

Prove: $\overline{AE} \cong \overline{BD}$

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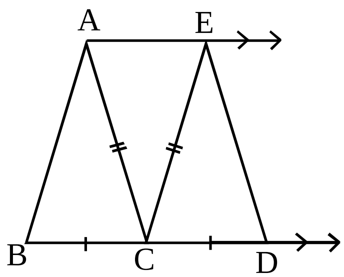
Congruence Review

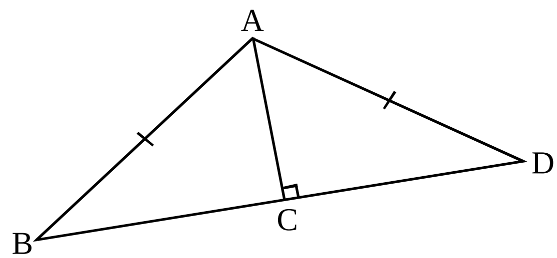
Geometry Re

Determine which of the following pairs of triangles are congruent and why: Triangles are **not** necessarily to scale.
Write ‘cannot be determined’ where appropriate.

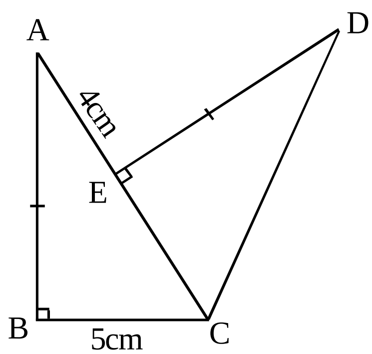
△ABC ≅ △____ by ____



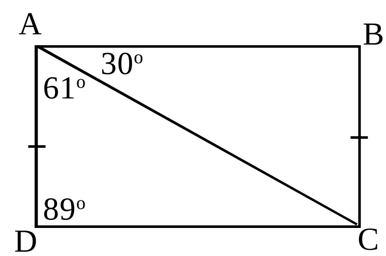
△ABC ≅ △_____ by ______



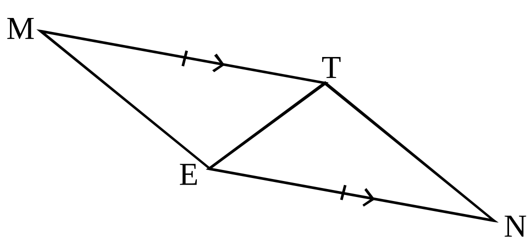
△CBA ≅ △_____ by ______



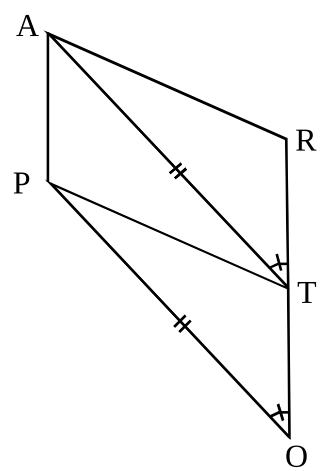
△ADC ≅ △______ by _____



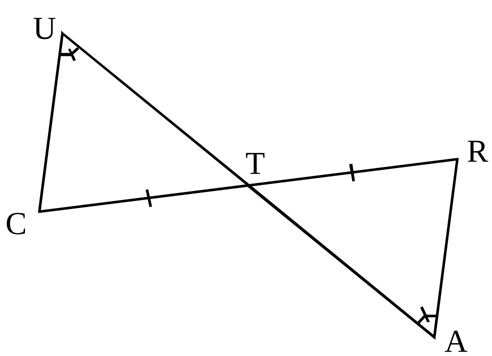
△MTE ≅ △______ by ______



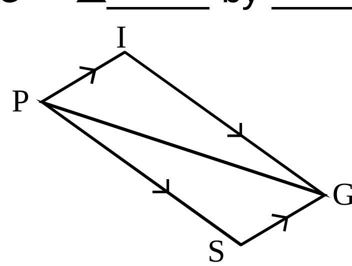
△PAT ≅ △______ by ______



△RAT ≅ △ ______ by _______



△PIG ≅ △______ by ______

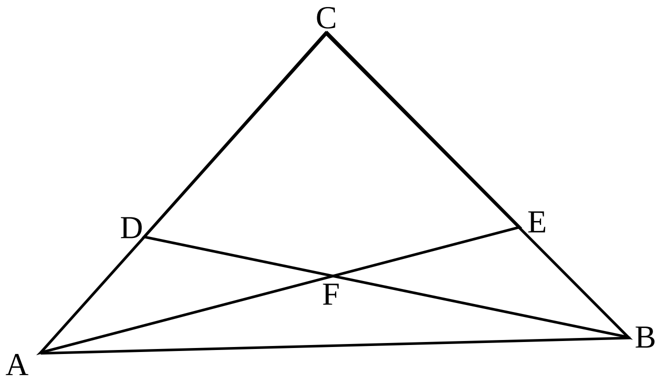


Name________________________ Period _____

CPCTC and Proofs

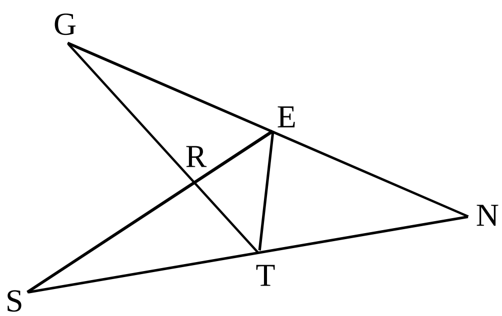
Geometry Re

Complete a proof for each: Use a separate sheet if needed.



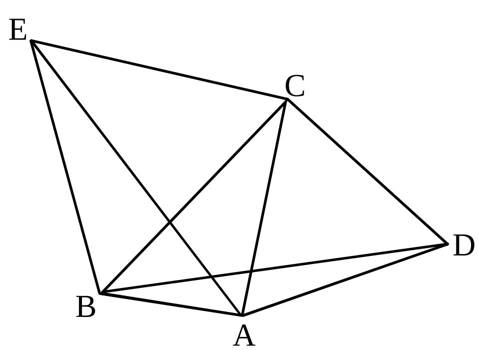
Given:
$\angle CDB \cong \angle CEA$
$\overline{CA} \cong \overline{CB}$
$\overline{AD} \cong \overline{BE}$

Prove: $\overline{AE} \cong \overline{BD}$



Given:
$\triangle GRE \cong \triangle SRT$
Prove: $\triangle SEN \cong \triangle GTN$

Challenge:
Triangles BCE and ACD are equilateral.
Prove that $\overline{AE} = \overline{BD}$.



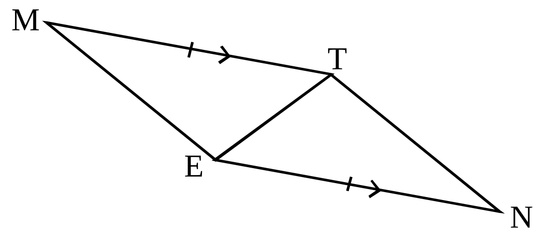
Name_________________________ Period _____

Congruence Review

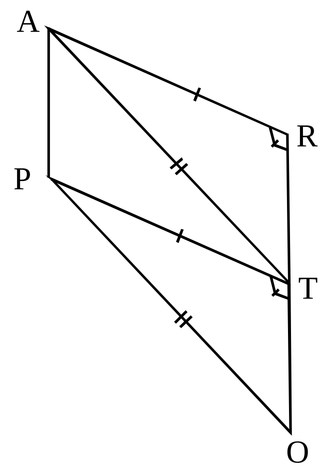
Geometry Re

Determine which of the following pairs of triangles are congruent and why: Triangles are **not** necessarily to scale.
Write 'cannot be determined' where appropriate.

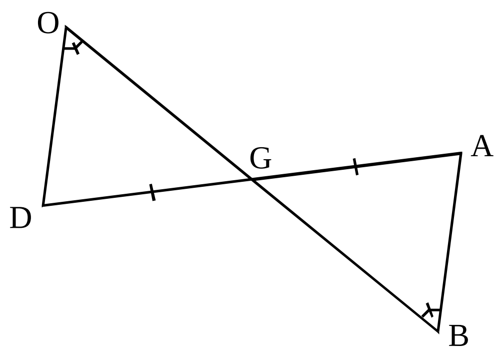
△MTE ≅ △______ by ______



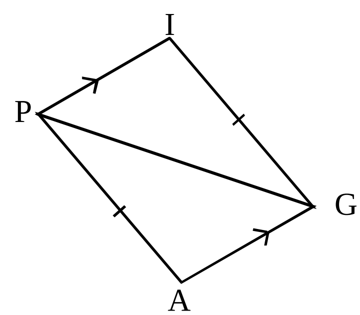
△PAT ≅ △______ by ______



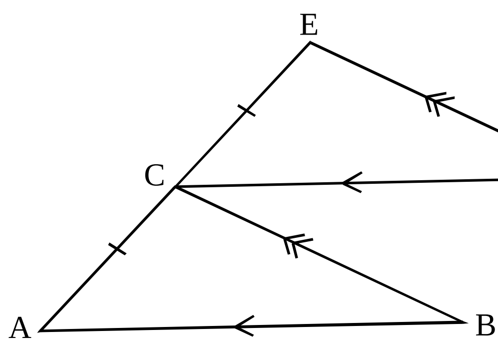
△DOG ≅ △ ______ by ________



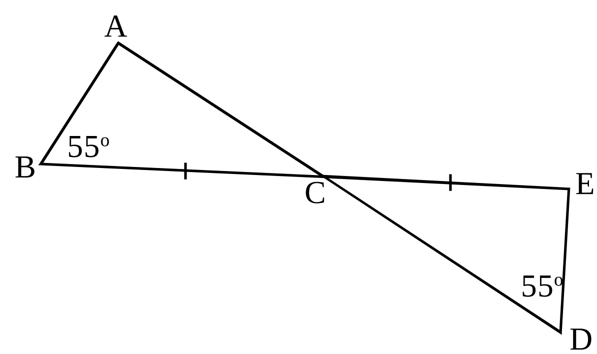
△PIG ≅ △______ by ________



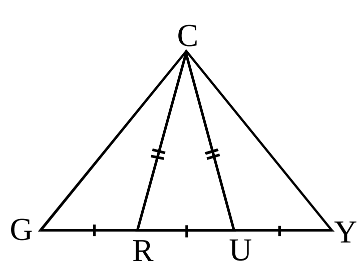
△ABC ≅ △______ by ______?



△BAC ≅ △______ by ______?



△CUG ≅ △ ______ by ________



Name________________________ Period _____

Proofs Practice

Geometry 4.8

For each of the following: Sketch the situation and label all given information. Create a two-column proof for the given statement.

1. Prove that in a given circle C, if chords AB and DE are congruent, then angles ACB and DCE are also congruent.

2. Segments AB and CD bisect each other. Prove that segments AD and BC are parallel.

Hints For Back:

1. You will need to use the base angles of an isosceles triangle.
2. You will use vertical angles.
3. You will prove two triangles that look congruent are congruent.
4. You will use Congruent Linear Angles.

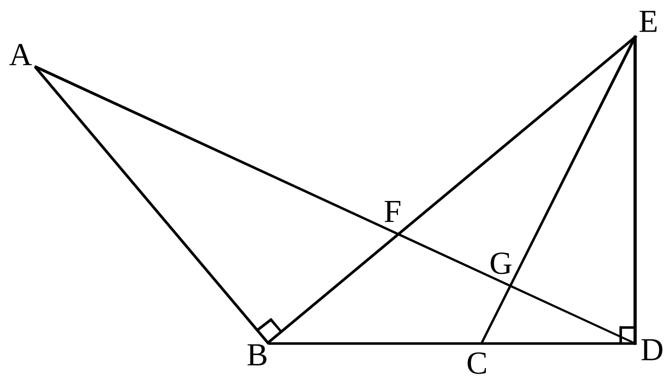
Name_________________________ Period ______

Proofs Practice

Geometry 4.8

Challenge: In the figure below, $\overline{AB} \cong \overline{BD}$ and EC bisects angle BED.

Prove AD ⊥ EC



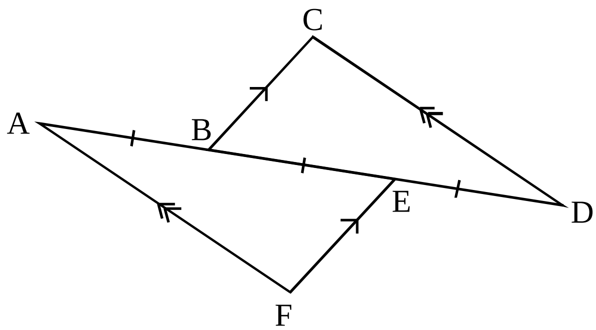
Hints can be found on the front.

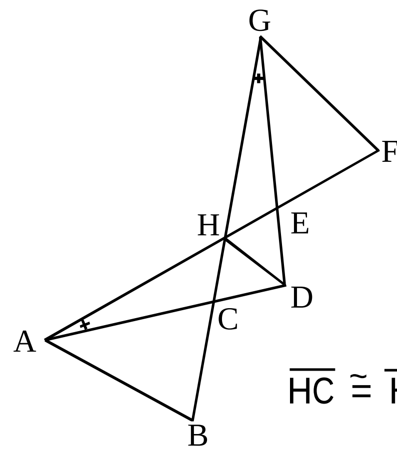
Test Review:

Geometry 4.8

Fill-in the blanks for each triangle congruence below.
Write 'cannot be determined' where appropriate.

△AEF ≅ △___ by ___?

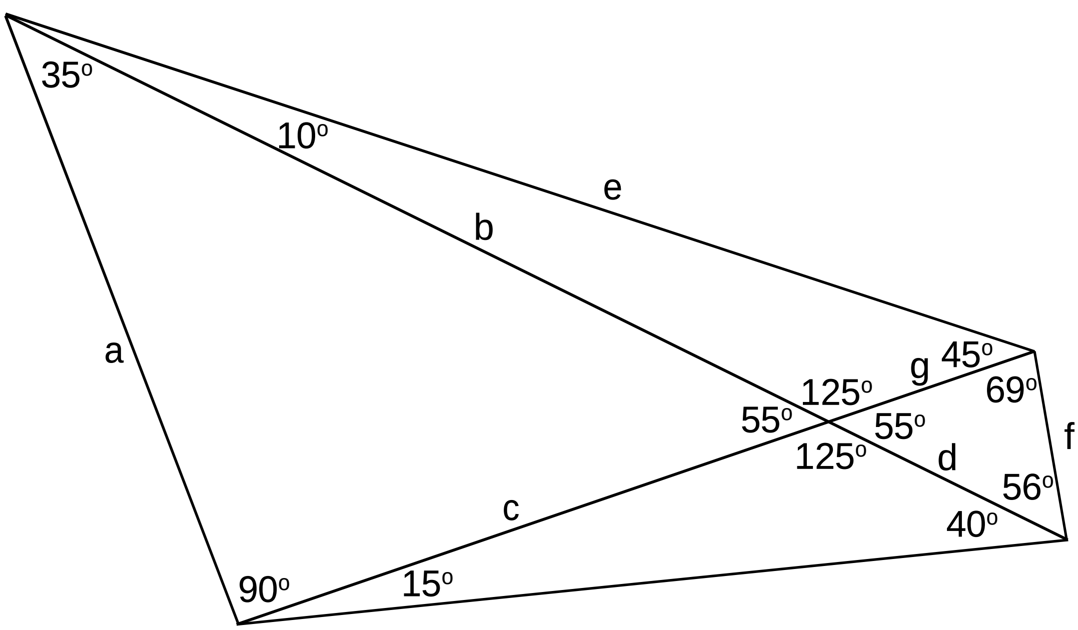




$\overline{HC} \cong \overline{HE}$

△HCA ≅ △___ by ___?
△AED ≅ △___ by ___?
△HCD ≅ △___ by ___?
△AHD ≅ △___ by ___?
△ABC ≅ △___ by ___?

List the sides below in order from shortest to longest. (not to scale)



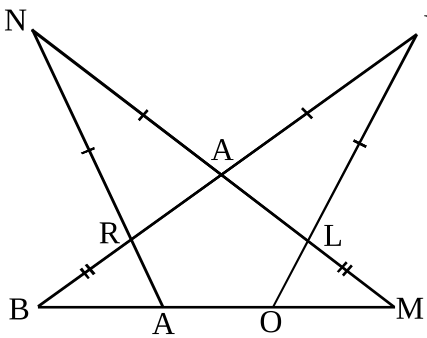
Name________________________ Period _____

Proofs Practice Test

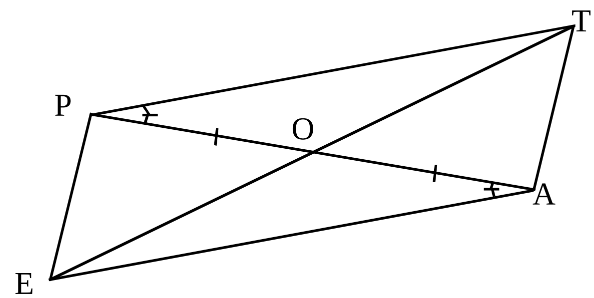
Geometry 4.8

Fill-in the blanks for each: Write “Cannot be determined” where appropriate.

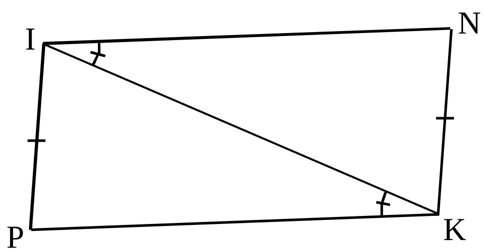
$\triangle$BOY ≅ $\triangle$ ______ by ________



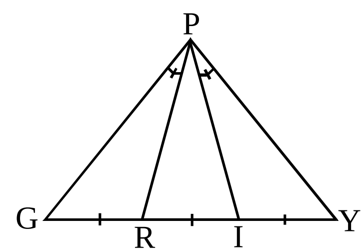
$\triangle$PAT ≅ $\triangle$______ by ______



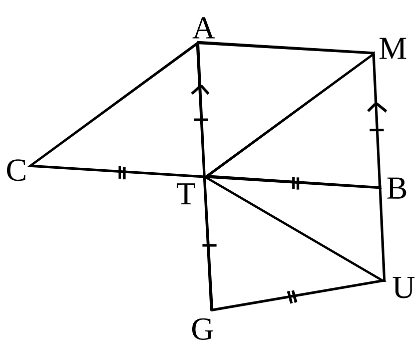
$\triangle$PIN ≅ $\triangle$ ______ by ________



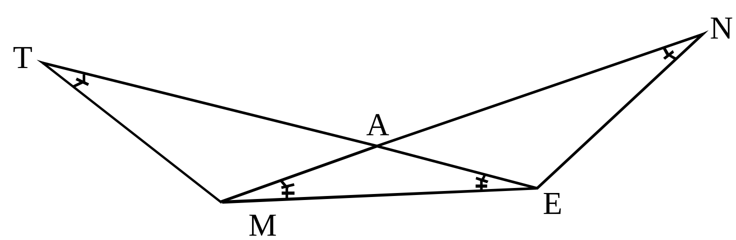
$\triangle$PIG ≅ $\triangle$______ by ________



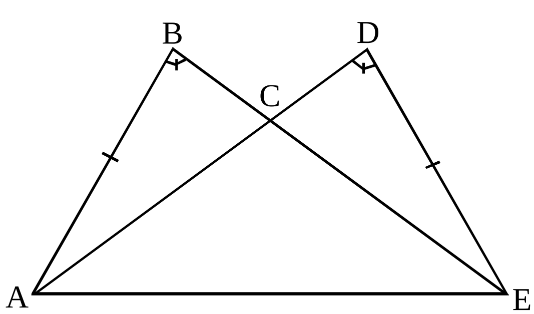
$\triangle$CAT ≅ $\triangle$ ______ by ________



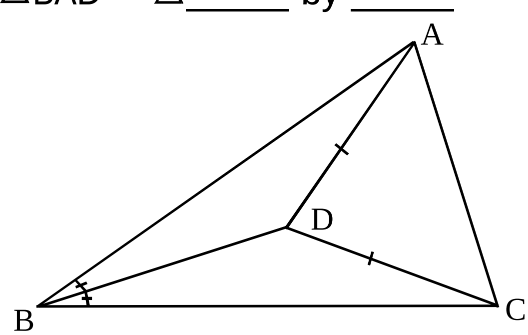
$\triangle$MTE ≅ $\triangle$______ by ______



$\triangle$ABE ≅ $\triangle$ ______ by ________



$\triangle$BAD ≅ $\triangle$______ by ______

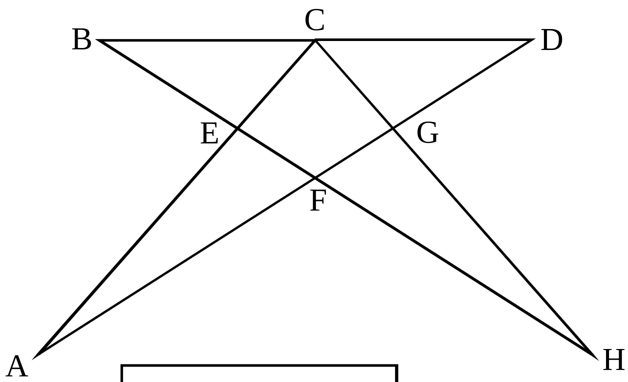


Name________________________ Period _____

Proofs Practice Test

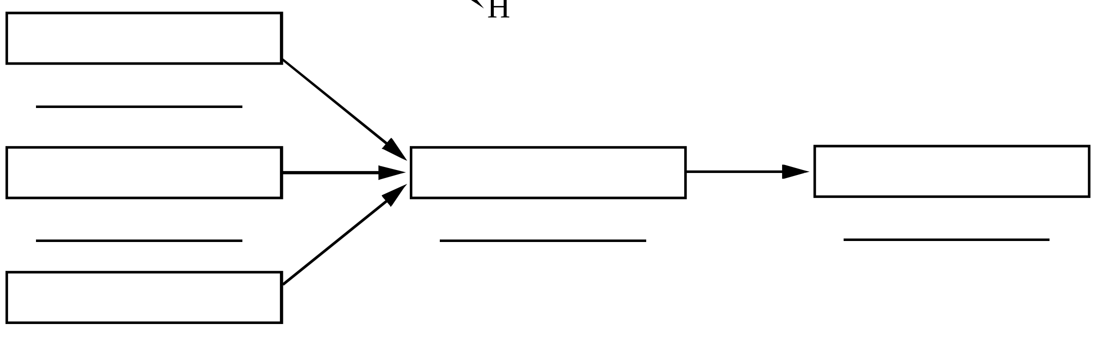
Geometry 4.8

Complete the flowchart proof below:



Given:
$\overline{BC} \cong \overline{DC}$
$\angle CEF = \angle CGF$
$\overline{CG} \cong \overline{CE}$

Prove: $\overline{CH} \cong \overline{CA}$



Write a two-column proof to prove that if congruent segments AB and CD are parallel, then AC = BD. Include a small sketch and all givens. Use as few spaces as possible.

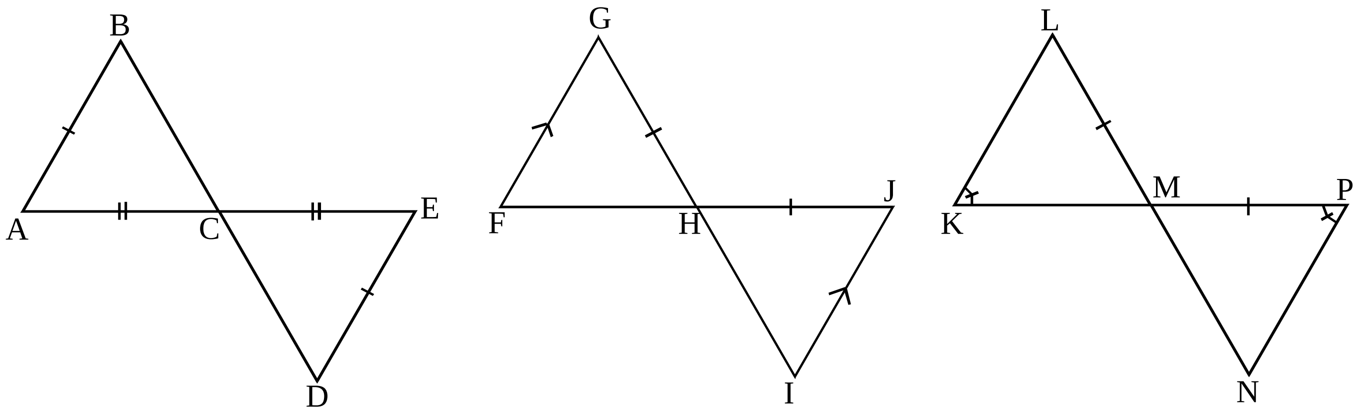
note: AC<AD

Test Review

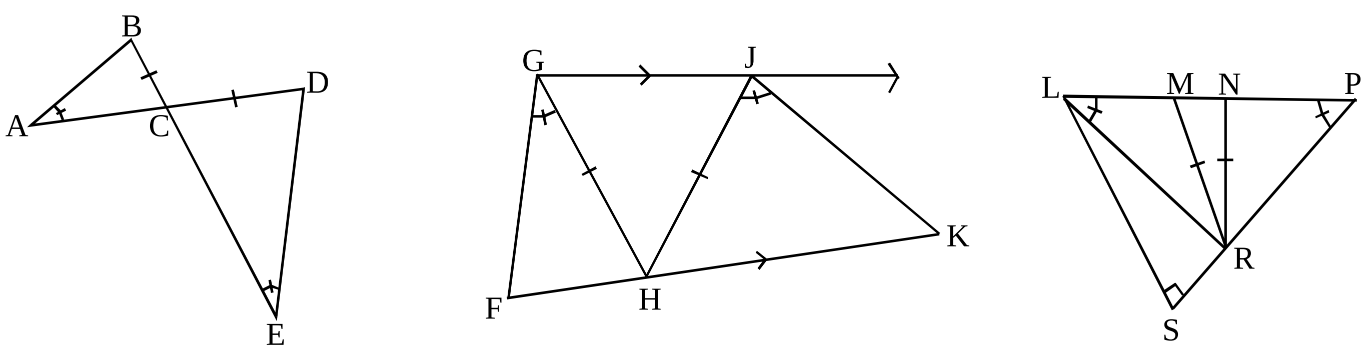
Geometry 4.8

Don't Guess!
The diagrams below are misleading and force you to ONLY USE WHAT YOU KNOW. Just because two triangles look congruent does not mean that they can be proven congruent using ASA, SSS, SAS, AAS, etc.

Practice:
Which pair(s) of triangles is congruent?

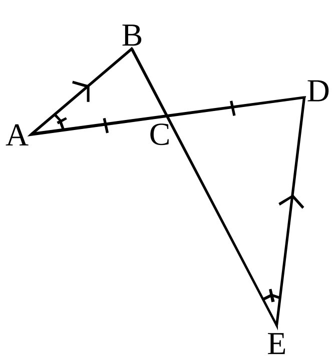


Practice:
Which pair(s) of triangles is congruent?

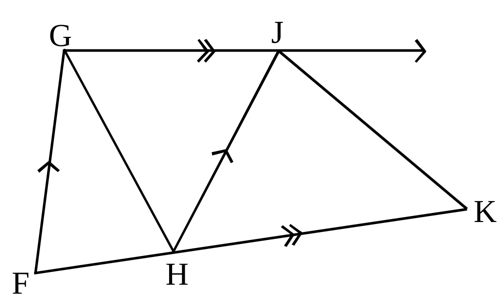


Practice:

Prove: C is the midpoint of BE.



Prove: ∠GJH ≅ ∠GFH



Name_________________________ Period _____

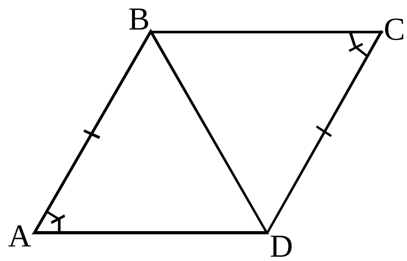
Test Review

Geometry 4.8

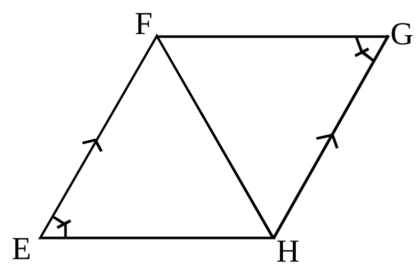
Practice: Misleading diagrams.

Which pairs of triangles are congruent AND why?

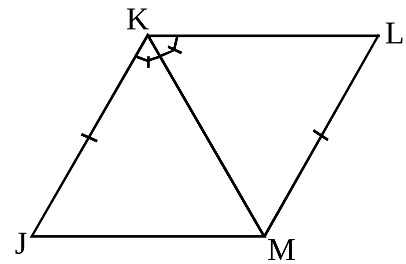
1. ______________



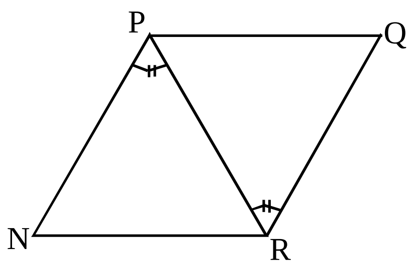
2. ______________



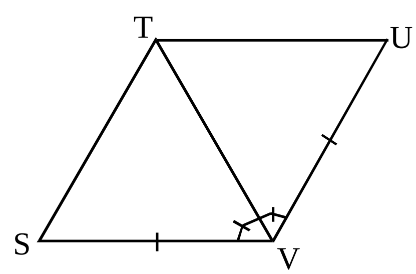
3. ______________



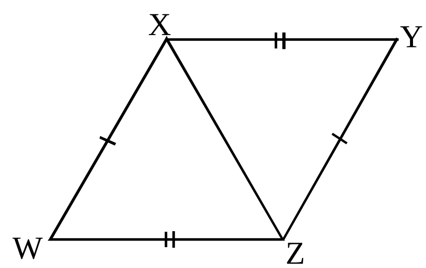
4. ______________



5. ______________



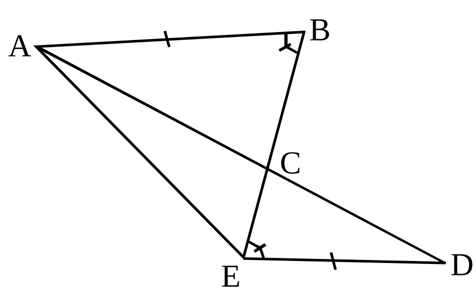
6. ______________



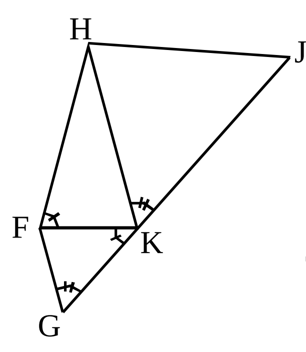
Practice: Misleading Diagrams.

Which pairs of triangles are congruent AND why?

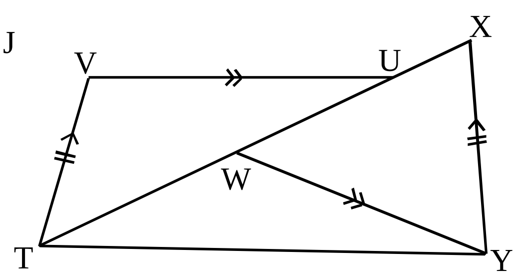
7. ______________



8. ______________



9. ______________



Practice:

On back, write a 2-column proof to show that the diagonals of a kite are perpendicular to each other.