Rational/Irrational

The Real Number System:

Whole Numbers: 0, 1, 2, 3, 4, ...

Integers: Positive and negative whole numbers: ... -2, -1, 0, 1, ...

Rational Numbers: Can be expressed as a fraction. This includes all repeating or terminating decimals.

\[
\begin{array}{cccc}
\text{Ex.} & 0.23 & \frac{1}{3} & 2.25 & \frac{5}{7} & 17 \\
\end{array}
\]

Irrational Numbers: Cannot be expressed as a fraction with integer numerator and denominator. This includes pi and most square roots (that cannot be simplified into fractions).

\[
\begin{array}{cccc}
\text{Ex.} & \pi & \frac{\sqrt{13}}{3} & 2.12345... & \sqrt{23} \\
\end{array}
\]

Practice:
Place each number where it belongs in the chart below:

\[
\begin{array}{cccccc}
\pi & 6 & 5.5555... & \sqrt{25} & \sqrt{53} & \frac{\pi}{3} \\
\frac{2}{2} & \frac{3}{3} & & & & \\
\end{array}
\]

Cube Roots:
A cube roots works similarly to a square root. If \( a = b^2 \), then \( b \) is a square root of \( a \). If \( a = b^3 \), then \( b \) is a cube root of \( a \).

Some are rational: \( \sqrt[3]{27} = 3 \), \( \sqrt[3]{125} = 5 \), and \( \sqrt[3]{1000} = 10 \).

Some are irrational: \( \sqrt[3]{20} = 2.714... \), \( \sqrt[3]{85} = 4.396... \), and \( \sqrt[3]{2} = 1.259... \).
Rational/Irrational

Practice:
Find the value of $x$ to the nearest thousandth, then classify the numbers below with all that apply: rational, irrational, integer, and/or whole number.

1. $x = \sqrt{20}$
2. $x = \sqrt[3]{64}$
3. $x = 2 + \sqrt{3}$

4. (right triangle)

Area:
The term square root comes from the fact that if you know the area of a square, the square root of the area is the side length. For example, if a square has an area of $25\text{cm}^2$, then the sides must be 5cm long.

Practice: Find the length of the sides of each square (round to the nearest thousandth where necessary) based on the given areas.

1. Area: $169\text{cm}^2$
2. Area: $19\text{cm}^2$
3. Area: $2.25\text{cm}^2$

Practice:
Can you find the edge length of each cube below given the volume?
Write each number below in the appropriate part of the diagram. DO NOT change the way the numbers are written, leave them as they are.

\[
\begin{array}{cccc}
\frac{\sqrt{2}}{2} & 25 & 4.567... & 2.5 \\
\frac{12}{12} & \sqrt{125} & \pi & 3.14
\end{array}
\]

1. The square root of a number is equal to 5.5. What is the number?  
1. ______________

2. The area of a square is 150cm². The side length of the same square is between what two integers?  
2. ______________

3. What is the length of the diagonal of a square that has 3-inch sides? Round your answer to the thousandth.  
3. ______________

4. On a test question, Mick was asked to find the square root of a number. Instead, he squared the number and got 49. What is the correct answer to the test question? Round your answer to the thousandth.  
4. ______________

5. How many of the numbers from 1 to 100 have a square root that is an irrational number?  
5. ______________
Irrational Numbers

Think you can learn this material without practicing? THAT is irrational.

Practice:
Find the side length of each square below based on the given area. Round to the thousandth where necessary.

6. 

| Area: 144cm² |

7. 

| Area: 22cm² |

8. 

| Area: 5.76cm² |

Practice:
Find each cube root. ALL of these answers are rational.

9. \( \sqrt[3]{1.331} \)

10. \( \sqrt[3]{195,112} \)

11. \( \sqrt[3]{-27} \)

Solve:

12. The volume of a cube is 64cm³. What is the length of the cube’s edges?
   
   12. ________________

13. A square has 10cm sides. The diagonal length of the same square is between what two integers?
   
   13. ________________

14. Order these three values from least to greatest: \( \sqrt[3]{1.5}, 1.12345..., \sqrt[3]{1.4} \).

   14. ________________

15. Solve: \( 1^3 + 2^3 + 3^3 + 4^3 + 5^3 - (1 + 2 + 3 + 4 + 5)^2 = \)

   15. ________________
Ordering Rationals and Irrationals

To be able to order rational numbers, you must be able to order decimals.

Example: Place the following decimals in order from least to greatest.

\[ 5.\overline{3} \]
\[ 5.3456... \]
\[ 5.25 \]
\[ 5.\overline{25} \]

Practice:
List each set in order from least to greatest.

1. \[ 41.2 \quad 41.\overline{2} \quad 4.1234... \quad 40.9 \]
2. \[ 9.4 \quad 9.345 \quad 9.543 \quad 9.34 \]
3. \[ \sqrt{45} \quad 6.7\overline{8} \quad 6.78\overline{9} \quad 6\frac{7}{9} \]

To order a list of numbers, it is easiest to write them all in the same form (usually decimal form) first. Line-up the decimal points and compare.

Example: Order from least to greatest.

\[ \sqrt{40} \quad 6.32455 \quad 6.29960 = \frac{3}{\sqrt{250}} \]
\[ 6.321 \quad 6.32100 \quad 6.32100 = 6.321 \]
\[ \text{becomes} \quad \text{and we order these:} \]
\[ 6.321 \quad 6.32132... \quad 6.32132... = 6.321 \]
\[ \sqrt{250} \quad 6.29960 \quad 6.32455 = \sqrt{40} \]
Ordering Rationals and Irrationals

Practice:
List each set in order from least to greatest.

1. 4.5  $\frac{17}{4}$  4.4  $\sqrt{20}$

2. $\frac{\sqrt{2}}{2}$  0.70  0.7  $\sqrt[3]{0.35}$

Practice:
List the values $w$ through $z$ from least to greatest (all figures are squares).

- Area: $120\text{cm}^2$
  w
- $x$
  8cm
- Area = y
  3.3cm
- z
  5cm  5cm
Ordering Numbers

Practice:
Order each set of three numbers from least to greatest.

1. __________ 2. __________ 3. __________
   A. \( \sqrt{25} \)  A. 4.567...
   B. \( \sqrt{10} \)  B. \( \frac{2}{5} \sqrt{94} \)
   C. 3.1  C. \( \sqrt{21} \)

4. __________ 5. __________ 6. __________
   A. \( \frac{\sqrt{2}}{2} \)  A. \( 97 \frac{1}{3} \)
   B. \( \sqrt{0.85} \)  B. \( \sqrt{9,475} \)
   C. \( \frac{1}{\sqrt{2}} \)  C. 68\( \sqrt{2} \)

Place each value below in its approximate place on the number line drawn:

7. A. \( \frac{27}{7} \)  B. \( \sqrt{19} \)  C. \( \frac{3}{\sqrt{14}} \)  D. \( 1 - \sqrt{2} \)  E. \( \frac{2\sqrt{7}}{3} \)  F. \( \sqrt{1 + \sqrt{64}} \)

8. A. \( \frac{7}{13} \)  B. \( 1 - \sqrt{3} \)  C. \( \frac{3}{\sqrt{0.73}} \)  D. \( \frac{\sqrt{2}}{9} \)  E. \( \frac{\sqrt{5} - 4}{3} \)  F. \( \sqrt{1 - \sqrt{2} - \sqrt{3}} \)