MOCK AMC 12

ADMINISTERED SEPTEMBER 2009

CREATED BY ***gfour84*** *WITH ASSISTANCE FROM* ***Caelestor;***

SPECIAL THANKS TO $\mathbf{1 = 2}$

This exam consists of 25 questions. Five choices are given per problem; only one is correct. A correct answer is worth 6 points, an unanswered question is worth 1.5 points, and an incorrect answer is worth no points.

You will have 75 minutes to complete this test. No aids of any kind except scratch paper, graph paper, ruler, compass, and protractor are permitted. Make sure you follow all instructions prior to, during, and after testing. Refer to the official AoPS threads for more information.

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You must turn the test into ***BOTH*** *gfour84 and Caelestor. Only the answers are required; you do not need to submit work. Remember your time limit! You should be treating this like the actual test! To submit answers, PM both gfour84 and Caelestor, or email us both at king-of-cards@cox.net and caelestor@aol.com (or caelestor@gmail.com). Failure to follow these instructions may delay all participants' results. HAVE FUN!!!!!*

1. What is $1-2-4+5+7-8-10+11+...+2009+2010$?

(A) less than -1 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** more than 1

2. In base 3, $N = 111000111000111000$. Determine the base-10 sum of the digits of the base 9 representation of N.

(A) 19 **(B)** 21 **(C)** 22 **(D)** 24 **(E)** 25

3. The stock market fluctuates a lot. On odd-numbered days, it loses 10% of its index from the preceding day; on even-numbered days, it increases 10% from the preceding day. To the nearest integer, what is the net change of the market between the start of September 1st and the end of September 10th?

(A) -6% **(B)** -5% **(C)** 0% **(D)** 5% **(E)** 6%

4. The mocking AMC is legendary for its difficulty; you only have a $\frac{1}{75}$ chance of passing it and proceeding onto the AIMEtator. If there are only two mocking AMCs offered this year, then the probability that you will attend the AIMEtator is $\frac{p}{q}$, where p and q are relatively prime. What is p?

(A) 141 **(B)** 143 **(C)** 145 **(D)** 147 **(E)** 149

5. The prices of seven rainbow-colored balls form an arithmetic progression, with red cheapest and indigo most expensive. The four cheapest balls cost \$0.50, and the four most expensive cost \$0.86. George has \$1 and buys as many balls as he wants. What is the minimum amount of money that he can have left over?

(A) \$0 **(B)** \$0.01 **(C)** \$0.02 **(D)** \$0.03 **(E)** \$0.04

6. If $\cos\theta = \frac{1}{x}$ for some angle θ and $\cos(2\theta) = x - 1$, then $x = \sqrt[a]{b}$ for integers a and b. The minimum value of ab is

(A) 4 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 16

7. Let $\sum_{k=1}^{2008} k^2 = p$ and $\sum_{k=1}^{2009} k^3 = q$. How many primes divide both p and q?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

8. Consider a thirteen-sided polygon with vertices that are all lattice points when graphed in the Cartesian plane. Its minimal possible area is $\frac{m}{n}$ for relatively prime integers m and n. Compute mn.

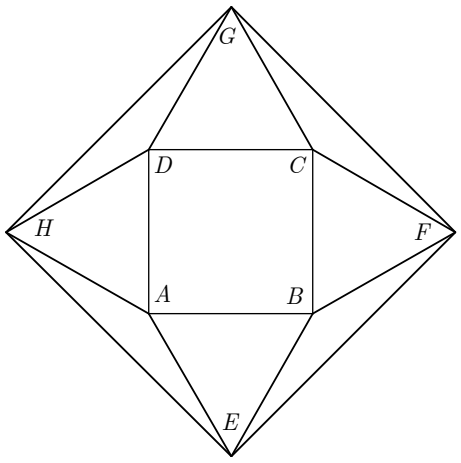
(A) 22 **(B)** 26 **(C)** 30 **(D)** 34 **(E)** 38

9. A quadratic f with real coefficients has root $2+3i$. Reflect f over $y=6$, shift it two units left, and three units up to obtain f'. The sum of the roots of $f'(2x)$ is

(A) -4 **(B)** -2 **(C)** 0 **(D)** 2 **(E)** 4

10. Examine the figure below. If $AC=5$, then what is the area of $EFGH$, given that $ABCD$ is a square and GDC, CFB, BEA, and AHD are equilateral?

(A) $\frac{25}{2}+\frac{25\sqrt{3}}{4}$ **(B)** $25+\frac{25\sqrt{3}}{2}$ **(C)** $\frac{20}{\sqrt{2}}+\frac{20\sqrt{3}}{\sqrt{2}}$ **(D)** $\frac{25}{\sqrt{2}}+\frac{25\sqrt{3}}{\sqrt{2}}$ **(E)** $25+\frac{25\sqrt{3}}{4}$



11. If $3f\left(\frac{1}{x}\right)+f(2x)=\log_2 2x$, determine $f(32)$.

(A) $\frac{13}{8}$ **(B)** $\frac{11}{8}$ **(C)** $-\frac{7}{8}$ **(D)** $-\frac{13}{8}$ **(E)** $-\frac{17}{8}$

12. A target board is made from five concentric circles, forming 4 rings that surround the center bullseye. The width of each ring and the diameter of the bullseye are identical. Millinova keeps throwing darts at the board until she hits the bullseye. Flarien does so as well independently of Millinova. Every dart successfully hits the board, but it is completely random where exactly each hits. What is the probability that Millinova used the same amount of darts as Flarien?

(A) $\frac{1}{49}$ **(B)** $\frac{1}{81}$ **(C)** $\frac{1}{111}$ **(D)** $\frac{1}{161}$ **(E)** $\frac{1}{199}$

13. Find the sum of all positive two-digit integers x such that $(x+2)(x^2+7x+13)$ is divisible by 9.

(A) 565 **(B)** 885 **(C)** 1065 **(D)** 1335 **(E)** 1605

14. Right triangle ABC with right angle C has the property that one of the trisection points D of segment AB is the foot of the altitude from C. If $BD < AD$, then find $\sin \angle CAB$.

(A) $\frac{1}{\sqrt{2}}$ **(B)** $\frac{1}{\sqrt{3}}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{\sqrt{5}}$ **(E)** $\frac{1}{\sqrt{6}}$

15. Let $f\left(\sqrt[3]{54+2\sqrt{675}}-\sqrt[3]{10+\sqrt{108}}\right)=a+\sqrt{b}$. Calculate $a+b$ if $f(x)=2x+\frac{1}{x+f(x)}$.

(A) 11 **(B)** 13 **(C)** 15 **(D)** 17 **(E)** 19

16. Let $z_k=\cos\left(\frac{2k\pi}{9}\right)+i\sin\left(\frac{2k\pi}{9}\right)$ for integer k. What is $(3-z_2)(3-z_4)\cdots(3-z_{16})$?

(A) 9841 **(B)** 9847 **(C)** 9848 **(D)** 9851 **(E)** 9853

17. An incredibly large population of $2009^{2009^{2009}}$ amoebas splits into groups. Every group has 42 amoebas, except for one, which contains only

(A) 7 **(B)** 14 **(C)** 21 **(D)** 28 **(E)** 35

18. Call a number *modular* if it is not divisible by any prime except for 5, 7, and/or 11 (e.g. 175 and 385 but not 363). What is the sum of all reciprocals of *modular* numbers?

(A) $\frac{576}{385}$ **(B)** $\frac{1161}{385}$ **(C)** $\frac{241}{240}$ **(D)** $\frac{29}{48}$ **(E)** $\frac{77}{48}$

19. A spaceship is at $(0,0,0)$. Every time it teleports, it moves to a lattice point one unit away. Let m be the number of paths the spaceship can take so that it takes 12 teleportations to reach $(4,4,4)$, while avoiding the black holes at $(1,1,1),(2,2,2),(3,3,3)$. Compute the sum of the digits of m.

(A) 9 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 21

20. Consider a sequence defined as follows:

$$\begin{aligned} F_1 = F_2 &= 1 \\ F_{n-2} + 2F_{n-1} &= F_n \text{ for } n > 2 \end{aligned}$$

Let $\sum_{n=1}^{\infty} \frac{F_n}{10^n} = R/G$ for relatively prime integers R and G. Compute $R + G$.

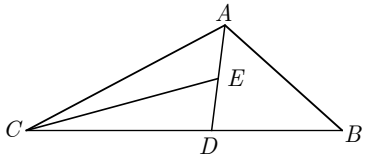
(A) 86 **(B)** 87 **(C)** 88 **(D)** 89 **(E)** 90

21. $ABCDEF$ is a regular hexagon with side length 1, with point G randomly located on the hexagon's boundary or in its interior. What is the maximum value of $AG^2 + BG^2 + CG^2 + DG^2 + EG^2 + FG^2$?

(A) 6 **(B)** 9 **(C)** 12 **(D)** 15 **(E)** 18

22. In the following figure, AD is the angle bisector to side BC of triangle ABC and E is the midpoint of AD. If $AC = 10$, $AB = 6.5$, and $BC = 10.5$, compute the length of CE^2.

(A) $\frac{7330}{121}$ **(B)** $\frac{7340}{121}$ **(C)** $\frac{7350}{121}$ **(D)** $\frac{7360}{121}$ **(E)** $\frac{670}{11}$



23. Consider the cubic $x^3 - x^2 + 3x + 4 = 0$ with roots a, b, c. Let $w = \tan^{-1} a + \tan^{-1} b + \tan^{-1} c$. Suppose the positive value of $\sec(w) = \frac{\sqrt{m}}{n}$ where m is not divisible by the square or any prime. Calculate $m + n$.

(A) 29 **(B)** 31 **(C)** 35 **(D)** 37 **(E)** 41

24. Suppose a, b, c are positive integers with a odd. Compute $a + b + c$, if

$$\begin{aligned} 2b - 6a + 8c + a + 2b - 5ab &= -396, \\ 8bc + 2b^2 - 3a^2 + 4ac &= 887. \end{aligned}$$

(A) 14 **(B)** 21 **(C)** 25 **(D)** 32 **(E)** 43

25. If $6 \tan^{-1} x + 4 \tan^{-1}(3x) = \pi$, compute x^2.

(A) $\frac{15 - \sqrt{3}}{33}$ **(B)** $\frac{15 - 8\sqrt{3}}{37}$ **(C)** $\frac{15 + 8\sqrt{3}}{37}$ **(D)** $\frac{15 - 8\sqrt{3}}{33}$ **(E)** $\frac{15 + 8\sqrt{3}}{33}$

Afterthoughts

We will offer additional mock AMC's before the season is out (or begins rather). Being just novices in a giant world of mathematics, we want input from everyone who took our test and either (1) found fault with it, or (2) want to see different variations in the questions. So, let us know what you think!!! We always want to make ourselves better at what we do – something which can help everyone. So, if you need practice in a certain area, or want something different in the next test, say so below.

Mini-Survey

(1) On a scale of 1 to 10, how hard was this test (10 being comparable to a mocking AMC)?

(2) On a scale of 1 to 10, how helpful was this test at delivering interesting concepts, regardless of difficulty (with 10 being very useful)?

(3) On a scale of 1 to 10 how imbalanced was this test (*i.e.* were there too much of one type of question and not enough of another, specifically Geometry vs Algebra vs Trig)? Note 10 means not at all.

(4) Were the questions clear and concise? Were you above to figure out what the question was asking, or how to go about answering the question (regardless of whether or not you solved it correctly and/or got the correct answer)?

(5) Were there any particular questions you felt were excellent? Too long and time-consuming? Too difficult? Your responses will enable us to create a revised version of this Mock AMC 12 9/09 for use by future problem solvers!

(6) If you would like us to use your name/AoPS screen name in tests, let us know.