MATHCOUNTS

2008-09
■ 42nd Mock Mathcounts ■
Team Round
Problems 1 - 10

State _____

Team

Members_____, Captain

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DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This round of the competition consists of ten problems which the team has 20 minutes to complete. Team members may work together to solve the problems. Team members may talk during this section of the competition. This round assumes the use of calculators, and calculations may also be done on scratch paper, but no other aids are allowed. The team captain must record the answers on his/her problem sheet, and all answers must be complete and legible. Only the team captain's problem sheet will be scored.

Total Correct	Scorer's Initials

1) If $a \neq b$ and
$$a^3 + a^2b + ab^2 + b^3 = \frac{a^4}{a - b}$$
Find b.

1)_____

2) Choose a positive integer. Replace the number with the sum of the squares of its digits and repeat this process until the number equals 1 or continues to cycle without including 1. If the number reaches 1 in this process, it is called a happy number. Find the fourth happy number.

2)_____

3) Alex is walking on a moving walkway, in the same direction as the walkway, while Richard is walking on a moving walkway in the opposite direction as the walkway. The walkways are both 50 feet long and move at the same speed. Alex and Richard start walking at the same time, at the ends of the walkways, and get to their respective ends at the same time. Given that Alex only needs to walk 10 feet by himself (as in, without counting the walkway), how much does Richard need to walk by himself?

3)_____

4) Consider two externally tangent circles, A and B. Let a line that is an external tangent to both of these circles intersect A and B at C and D, respectively. Let E be the circle tangent to A, B, and CD. If the radius of circle A is 9 and the radius of circle B is 16, what is the radius of circle E? Express your answer as a common fraction.

4)_____

5) If $-3 \leq a, b, c \leq 3$, what is the probability that $|a| + |b| + |c| \leq 1$? Express your answer as a common fraction.

5)_____

6) Six people get their math tests passed back to them. If each person gets a test that is not their own, in how many ways could the tests have been passed out?

6)_____

7) Ray arbitrarily decides to flip a coin 15 times. He flips 7 heads and 8 tails. When you randomly arrange the 7 heads and 8 tails in a row, how many times would you expect that a head appears next to a tail? For example, if you have 7 heads and then 8 tails, a head appears next to a tail once. Express your answer as a decimal to the nearest tenth.

7)_____

8) Consider the factorials of all the integers from 1 to 121 inclusive. When the factorials are multiplied and the result is expressed in base 11, how many zeros does this number end in?

8)_____

9) Consider a square of side 1 with all four vertices as centers of circles with radius 1. Find the area common to at least 2 of the circles. Use 3.14 for π, and express your answer as a decimal to the nearest hundredth.

9)_____

10) Let ABC be an equilateral triangle with $AB = 6$. Let D be the midpoint of AB and let C be the midpoint of AE. Let F be the intersection of DE and BC. In addition, let G be the midpoint of AD, H be the point on AC such that $AH = \frac{3}{4}AC$, and I be the intersection of GE and DH. The ratio of the area of $AGIH$ to the area of $ADFC$ is $\frac{m}{n}$, where m and n are relatively prime integers. Find $m + n$.

10)_____