MATHCOUNTS

2008
■ Yongyi's National Competition ■
Sprint Round
Problems 1–30

Name ____

State ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books, or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

1. In a field of 108 aliens, each alien has either 5 heads or 103 heads. If I counted 1030 heads in total, how many aliens have 103 heads?

1. ______________ aliens

2. A supermarket advertised a sale of 75% off, and afterwards, the price of a pen was $4. What was the original price of a pen before the sale?

2. $______________

3. In a right triangle, the two legs have lengths of 65 and 72. What is the length of the hypotenuse?

3. ______________

4. In the expansion of $(x+1)^6$, what is the coefficient of x^3?

4. ______________

5. In a school, the ratio of boys to girls is 3 : 5. After 20 more boys attended the school, the ratio is now 1 : 1. How many girls are in the school?

5. ______________ girls

6. In a row of 10 seats, in how many ways can I choose two seats that are not next to each other?

6. ______________ ways

7. If I roll a standard 6-faced die three times, what is the probability that I roll at least one 6? Express your answer as a common fraction.

7. ______________

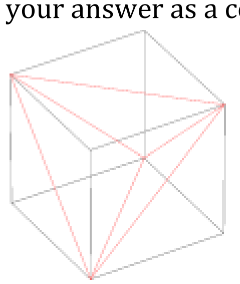
8. In a circle with radius 13, two parallel chords with length 10 are joined to create a rectangle. Find the area of this rectangle.

8. ________________

9. Find the number of real solutions to the equation $x^2 + 89x + 2008 = 0$.

9. ________ solutions

10. Find the volume of the largest regular tetrahedron that can be inscribed in a cube of side length 2, as shown below. Express your answer as a common fraction.



10. ________________

11. How many numbers from 1 to 20 inclusive must be on a blackboard before the probability of two of them being the same is greater than 1/2?

11. ________ numbers

12. I start at my house and flip a coin. If it lands heads, I walk exactly one meter east. If it lands tails, I walk exactly one meter west. Once I return home, I stop flipping coins. What is the probability that I return home after four flips? Express your answer as a common fraction.

12. ________________

13. Find the number of degrees the hour hand travels on a 12-hour clock between 1:23 P.M. and 6:11 P.M.

13. ________ degrees

14. Two trains are traveling toward each other at opposite ends of a 200-mile track. One travels at 60 miles per hour and the other travels at 90 miles per hour. If the slower train starts at 9:00 A.M. and the faster train starts at 9:30 A.M., at what time will the two trains collide?

14. ____________________

15. What is the probability that two points placed randomly in a unit circle will be less than one unit away from each other? Express your answer as a common fraction.

15. ____________________

16. How many isosceles triangles can be formed by connecting three vertices of a cube?

16. __________ isosceles triangles

17. How many three-digit numbers fit the property that the second digit is greater than or equal to the first digit and the third digit is greater than or equal to the second digit?

17. __________ numbers

18. Sally folds an 8.5×11 sheet of paper in half either horizontally or vertically, keeping the minimum possible perimeter, four times. What is the perimeter of the resulting rectangle? Express your answer as a decimal to the nearest thousandth.

18. ____________________

19. What is the y-intercept of the line perpendicular to $y = 2x + 3$ at the point (3, 9)? Express your answer as a common fraction.

19. ____________________

20. A sequence begins with 1, 4, 7, … Another sequence begins with 99, 95, 91, … At which position is the positive difference between the respective terms of the two sequences the minimum?

20. ____________________

21. Given that $x^2 + y^2 = 29$ and $xy = 10$, find the minimum value of $\frac{1}{x} + \frac{1}{y}$. Express your answer as a common fraction.

21. ____________________

22. The word value of a word is the sum of the values of its letters, where A = 1, B = 2, C = 3, D = 4, etc. For example, the word value of CAR is 3 + 1 + 18 = 22. How many four-letter "words" exist whose word value is 28? "Words" need not be real English words.

22. ______________ words

23. The rule in the Jungle of Nool is that every twenty years, each pink bunny gives birth to a brown bunny and every brown bunny gives birth to a pink bunny and a brown bunny. In 2000, the only pink bunny in the Jungle of Nool gave birth to a brown bunny. If bunnies die immediately after giving birth, how many bunnies will be born in the Jungle of Nool in 2100?

23. ______________ bunnies

24. If $(2x + 3y + 1)^{3x-2y-1}$ is not defined, find $x + y$. Express your answer as a common fraction.

24. ____________________

25. Given that $x^x = 3$, find the value of $2x^{2x}$.

25. ____________________

26. First, six cards are drawn from the top of a standard 52-card deck and placed flat on a table. Next, two cards are randomly chosen from those six cards. What is the probability that at least one of them will be a spade? Express your answer as a common fraction.

26. ____________________

27. A regular octagon is inscribed in a circle and another regular octagon is circumscribed about the same circle. Find the ratio of the area of the smaller octagon to the area of the larger octagon. Express your answer as a common fraction in simplest radical form.

27. ____________________

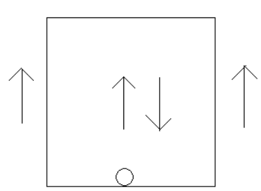
28. Two polygons, not necessarily distinct, are randomly chosen from a list of regular polygons of side length 1, from 3 sides to 12 sides, inclusive. What is the probability that they will tessellate the plane? Express your answer as a common fraction.

28. ____________________

29. A time is *reducible* if the hour and minute share a common factor greater than 1. For example, 9:15 is reducible because they share a common factor of 3, but 8:27 is not. How many reducible times are there between 12:00 A.M. and 12:00 P.M.?

29. __________ reducible times

30. An army of soldiers, which is a square measuring 50 feet by 50 feet, marches north at a constant rate of 1 foot per second. The mascot, a dog, starts at the south end of the square and runs at a constant speed to the north end and back to the south end. If this takes exactly 2 minutes, what is the rate at which the dog ran, in feet per second? Express your answer as a common fraction.



30. ______________ ft/sec

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2008
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Target Round
Problems 1 and 2

Name ____

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This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. Record only final answers in the designated blanks on the problem sheet. All answers must be complete, legible, and simplified to lowest terms. This round assumes the use of calculators, and calculations may also be done on scratch paper, but no other aids are allowed. If you complete the problems before time is called, use the time remaining to check your answers.

Total Correct	Scorer's Initials

1. Bob writes the list of numbers from 1 to 99 without breaks, starting with 12345678910111213… on a sheet of paper. Aidan writes the list of numbers from 99 to 1 directly underneath Bob's list, starting with 99989796959493… A scanner starts at the left edge of the paper and runs to the right. When the scanner finds the same digit in both lists, it beeps. How many times will the scanner beep before it reaches the right edge of the paper?

1. ______________ times

2. Seven identical coins are weighted such that when all of them are flipped, the probability of two of them being heads is twice the probability of three of them being heads. When one of these coins is selected and flipped once, what is the probability that it will come up heads? Express your answer as a common fraction.

2. ____________________

MATHCOUNTS

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Target Round
Problems 3 and 4

Name ____

State ____

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Total Correct	Scorer's Initials

3. How many lists consisting of seven numbers have the property that the median is 11, the mean is 12, and the mode is 13?

3. ________________ lists

4. In solving the quadratic equation $ax^2 + bx + c = 0$, Bob made an error, solved $bx^2 + cx + a = 0$, and got the roots $\frac{3+\sqrt{14}}{5}$ and $\frac{3-\sqrt{14}}{5}$. Alan made an error, solved $cx^2 + ax + b = 0$, and got the roots -1 and $5/6$. Xiaoyu made an error, solved $ax^2 + cx + b = 0$, and got the roots $-3 + \sqrt{14}$ and $-3 - \sqrt{14}$. What were the original roots of the equation? Express your answer as an ordered pair.

4. (______ , ______)

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2008

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Target Round

Problems 5 and 6

Name ____

State ____

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Total Correct	Scorer's Initials

5. In reducing the fraction $\frac{16}{64}$, Runpeng wasn't concentrating properly and canceled out the 6's to get 1/4. Miraculously, that was equivalent to the original fraction! Let fractions like this be called "interesting fractions." In addition, an interesting fraction cannot have 0's in its numerator or denominator and its value must be less than 1. How many interesting fractions have two-digit numerators and denominators? Note that the fraction does not have to be in simplest form after "reducing."

5. ____________ fractions

6. Three positive real numbers a, b, c satisfy the equation $a + b + c = abc$. What is the minimum possible value of $a^6 + b^6 + c^6$?

6. ____________________

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Target Round
Problems 7 and 8

Name ____

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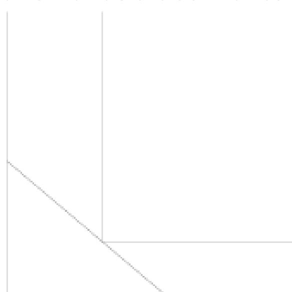
Total Correct	Scorer's Initials

7. How many sets containing three numbers exist that consist of distinct integers from 2 to 10 and whose elements all share a common factor greater than 1? The set {4, 6, 10} is an example of such a set, while {5, 7, 10} is not.

7. ________________ sets

8. A 10-foot wide hallway intersects a 6-foot wide hallway in the corner of a building. What is the maximum length, in feet, of a pole that could be carried past the corner without tilting it? Assume that the width of the pole is zero and express your answer as a decimal to the nearest hundredth.



8. ________________ feet