Solutions to Warm Up! ☺ Jenny

1. Draw ten circles, imagine these circles are the lollipops. Then draw 4 dividers at any place you want before, after, or in-between the circles. The number of circles between each divider is the number of lollipops per flavor. Here is a sample picture:

○ ○ | | ○ ○ ○ ○ | ○ ○ ○ | ○

This picture represents 2 orange lollipops, 0 berry lollipops (there are no circles in-between the second and third dividers), 4 pineapple lollipops, 3 pomegranate lollipops and 1 lime lollipop. As you can see, there are 14 objects, and 14 spots. The question asks us the number of combinations of lollipops. So, all we need to do is to find the number of ways we can put 4 dividers into 14 spots. Remember, the dividers are not distinct, so the answer is 14 choose 4, which is **1001 ways.**

2. Emily has to walk a total of 4 blocks south and 5 blocks east. A simplified version of this problem would be that there are 9 boxes and you have to put either a S (south) or an E (east) in each box for a total of 4 S's and 5 E's. The answer would then be 9 choose 4 (number of ways to put 4 S's, E's would go in the remaining spaces) or vice versa. The answer is **126 paths.**

3. The radius is one-half of the remaining length of the altitude of the triangle. This can be proven using 30-60-90 triangles, but we won't elaborate on it now. So, the altitude must be $3\sqrt{3}$. The Pythagorean theorem gives us the equation: $s^2= (s/2)^2+a^2$, where s is the side length and a is the altitude. This solves to $a=(s\sqrt{3})/2$. We know that the altitude, a, is $3\sqrt{3}$, so when we plug it in that gives us s=**6**.

4. Madison buys an outfit from a clothing retailer at an unknown price. Let's call that unknown price x. She then sells it for 60 percent more, which is $1.6x$. Since no one buys it, she lowers the price by 16 dollars, so the price is now $1.6x$-16. She again lowers the price by 25 percent, so the price is now .75($1.6x$-16). She then sells it with a profit of 30 dollars. That means that .75($1.6x$-16) is equal to x+30. This equation yields $\mathbf{x = 210}$ when solved. So Madison bought the outfit for **210 dollars**.

5. The expression $\sqrt{10 + 2\sqrt{21}}$ can be expressed as $\sqrt{(\sqrt{3})^2+(\sqrt{7})^2+2\sqrt{(7)(3)}}$ which simplifies to $\sqrt{(\sqrt{3}+\sqrt{7})^2}$ which simplifies to $\sqrt{3}+\sqrt{7}$. Therefore, 3 is a and 7 is b (or vice versa, it doesn't matter). The positive difference would then be **4**.

6. We know that ST=D. In this problem we have two ST=D cases. Since the distances are both the same we can write a $S_1T_1=S_2T_2$ equation. Let's call the speed of the wind w. The speed will be in mph and the times in hrs. In the first case, the speed is 1000-w and the time is 3. In the second case, the speed is 1000+w and the time is 1. The equation would be (1000-w)(3)=(1000+w)(1). Solving yields w=**500mph**.

7. The units digit of 2009^{2010} is 1. This is because powers of 2009 end in 9, 1, 9, 1, 9…etc. The 23rd triangular number can be found by the equation n(n+1)/2, and plugging in 23 for n gives us 276. The units digit of that is 6. So add the two units digits together to get **7**.

8. We can use Pascal's Triangle's relationship to binomials for this one.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

The first row I will call B0, the second row I will call row B1…the sixth row I will call row B5. Row B5 will help us with binomials to the fifth power. So, using Pascal's Triangle we get that the expansion of $(x+y)^5$ is $1x^5+5x^4y^1+10x^3y^2+10x^2y^3+5x^1y^4+1y^5$. (I put some unnecessary one's in there so you could see the relationship.) So, the coefficient of x^3y^2 is **10**.

9. This problem is actually one of those tricky logic problems. 214 of the 215 players must be eliminated to determine a single winner. Each game, one player is eliminated. Therefore, **214 games** must be played to eliminate 214 players and determine a single winner.

10. Since length AB is equal to length OD, and OD is a radius, the length AB is also equal to all other radii. Length BO is a radius, and is congruent to AB. That means that triangle ABO is isosceles, and by base angles of an isosceles triangle, angles BAO and BOA are congruent. Let's call the degree measure of angle BAO x. That means BOA is also x and ABO is 180-2x. By linear angles, OBE is 2x. Since OB and OE are both radii, triangle BOE is also isosceles. By base angles of an isosceles triangle, OBE and OEB are congruent, and both are 2x. Therefore angle BOE is 180-4x. Since angle EOD is 60, angle EOC must be 120 by linear angles. Angle EOC is equal to angle BOE plus angle BOA. We know BOE is 180-4x and BOA is x. So 180-4x+x is 120. Solving gives us x equals **20 degrees**. All of this is illustrated in the diagram below.